PRACTICE PAPERS

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PHYSICS for you

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Contents

editorial

Dropleton – A New Particle ?

According to the authors who have discovered microscopic particle clusters in solids, which behave like a liquid have the properties of a quasi particle. This particle has a very short life span. Stimulated by light, the smaller particles briefly condense into a 'droplet' with the characteristics of liquid water. This can have ripples. The life time of this droplet is only about 25 picoseconds (trillionth of a second).

This strangely behaves like a liquid. It is thought that five electrons are forming the new particle with five holes. The very short life-time of the particle, the changing positions of these particles (or electron-hole combinations) give it an appearance of a liquid drop.

In order to evaluate and appreciate the new aspects in this experiment, I am quoting what is given in the Penguin Dictionary of Physics.

"Exciton : An electron in combination with a hole in a crystalline solid. The electron has gained sufficient energy to be in an excited state and is bound by electrostatic attraction to the positive hole. The exciton may migrate through the solid and eventually the hole and electron recombine with emission of a photon".

Any new discovery should excite our students. Our advice to our research student is this: Appreciate what others have done and learn what other scientists have performed earlier. Making that as a starting point, one has to go further devising your own methods and extensions.

Anil Ahlawat
Editor

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / Other PMTs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their solutions. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

PROBLEM Set 22

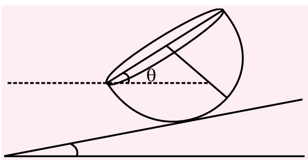
1. A 50 kg woman is on a large swing of radius 9 m that rotates in a vertical circle at 6 rev/min. What is the magnitude of her weight when she has moved halfway up?

(a) 522 N (b) 521 N
(c) 523 N (d) 520 N

2. A car is moving uniformly along a circle of radius $250\sqrt{3}$ meter with a time period of 20 seconds. A stone is thrown from the car such that it again lands on the car when the car is at a diametrically opposite point in the orbit. The angle of projection of the stone as seen from the ground should be (Take $g = 10$ m s^{-2})

(a) 45° (b) 60°
(c) 30° (d) none of these

3. A hemispherical shell rests on a rough inclined plane whose angle of friction is λ, then the maximum inclination of the plane of base of rim from the horizontal is

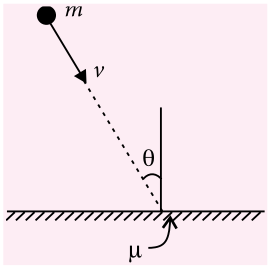


(a) $\theta = \sin^{-1}(\sin\lambda)$ (b) $\theta = \sin^{-1}(2 \sin\lambda)$
(c) $\theta = \sin^{-1}(3\sin\lambda)$ (d) $\theta = \sin^{-1}(4\sin\lambda)$

4. At NTP the density of a gas is 1.3 kg m^{-3} and the velocity of sound propagation in the gas is 330 m s^{-1}. The degree of freedom of the gas molecule is

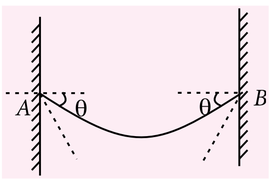
(a) 3 (b) 5
(c) 6 (d) 7

5. A small ball of mass m moving with a velocity v (just before the impact) strikes a rough surface of frictional coefficient at an angle θ with the vertical as shown in the figure. If the vertical component of velocity of the ball just after impact is found to be (v/2) in vertically upward direction then the horizontal velocity of the ball after impact in horizontal direction is



(a) $v\sin\theta$
(b) $v\sin\theta - \mu\{(v/2) - v\cos\theta\}$
(c) $v\sin\theta + \mu\{(v/2) + v\cos\theta\}$
(d) $v\sin\theta - \mu\{(v/2) + v\cos\theta\}$

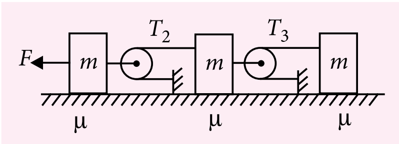
6. A heavy string of mass m hangs between two fixed points A and B at an angle θ with the horizontal as shown in the figure. The tension at the lowest point in the string is



(a) $mg/(2\sin\theta)$ (b) $mg/(2\cos\theta)$
(c) $mg/(2\tan\theta)$ (d) $mg/(2\cot\theta)$

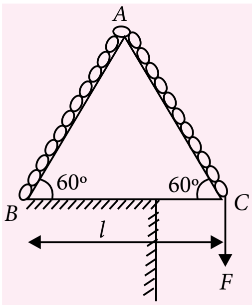
By Akhil Tewari, Author Foundation of Physics for JEE Main & Advanced, Senior Professor Physics, RAO IIT ACADEMY, Mumbai.

7. On a rough table, three blocks (including the first block) are placed as shown in the figure. Mass of each block is m and coefficient of friction for each block is μ. A force F is applied on the first block so as to move the system. The minimum value of F should be



(a) $8\ \mu mg$ (b) $9\ \mu mg$
(c) $7\ \mu mg$ (d) $5\ \mu mg$

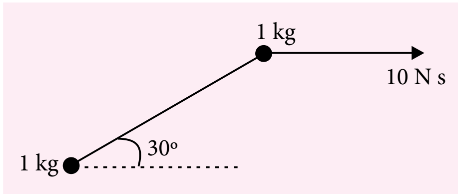
8. A fixed wedge ABC in the shape of an equilateral triangle of side l. Initially, a chain of length $2l$ and and mass m rests on the wedge as shown. The chain is slowly being pulled down by the application of a force F as shown. Work done by gravity till the time, the chain leaves the wedge will be



(a) $-\left(\frac{(\sqrt{3}+1)mgl}{2}\right)$ (b) $\left(\frac{(\sqrt{3}+2)mgl}{2}\right)$

(c) $-\left(\frac{(\sqrt{3}+2)mgl}{4}\right)$ (d) $\left(\frac{(\sqrt{3}+4)mgl}{4}\right)$

9. Two balls of masses 1 kg each are connected by an inextensible massless string. The system is resting on a smooth horizontal surface. An impulse of 10 N s is applied to one of the balls at an angle 30° with the line joining two balls in horizontal direction as shown in the figure. Assuming that the string remains taut after the impulse, the magnitude of impulse of tension is



(a) 6 N s (b) $\frac{5}{2}\sqrt{3}$ N s

(b) 5 N s (d) $\frac{5}{\sqrt{3}}$ N s

10. A skier travels with a constant speed of 6 m s^{-1} along a parabolic path $y = \frac{x^2}{20}$. Find the acceleration of the skier when he is at (10, 5). Neglect the size of the skier.

(a) $\frac{9}{10\sqrt{2}}$ units (b) $\frac{9\sqrt{2}}{10}$ units

(c) 1 units (d) None of these

JEE ADVANCED : PRACTICE PAPER

READY STEADY GO!!

EXAM ON 24TH MAY

PAPER-1

SECTION-1

One or More Than One Options Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONE or MORE THAN ONE are correct.

1. A particle of mass m moves on the x-axis as follows: it starts from rest at $t = 0$ from the point $x = 0$, and comes to rest at $t = 1$ at the point $x = 1$. No other information is available about its motion at intermediate times $(0 < t < 1)$. If α denotes the instantaneous acceleration of the particle, then

(a) α cannot remain positive for all t in the interval $0 \le t \le 1$.

(b) $|\alpha|$ cannot exceed 2 at any point in its path.

(c) $|\alpha|$ must be ≥ 4 at some point or points in its path.

(d) α must change sign during the motion, but no other assertion can be made with the information given.

2. A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is given by

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\dfrac{M\eta}{L}}$

(c) $2\pi\sqrt{\dfrac{ML}{\eta}}$ (d) $2\pi\sqrt{\dfrac{M}{\eta L}}$.

3. A voltmeter and an ammeter are joined in series to an ideal cell, giving readings V and A respectively. If a resistance equal to resistance of ammeter is now joined parallel to ammeter then

(a) V will decrease slightly

(b) V will increase slightly

(c) A will become half of its initial value

(d) A will become slightly more than half of its initial value

4. A charged particle goes undeflected in a region containing electric and magnetic fields. It is possible that

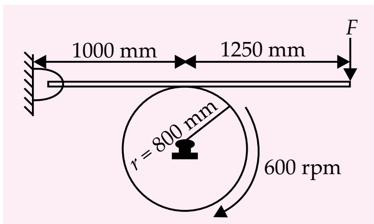
(a) $\vec{E} \,||\, \vec{B},\ \vec{v} \,||\, \vec{E}$

(b) $\vec{E}$ is not parallel to $\vec{B}$

(c) $\vec{v} \,||\, \vec{B}$ but $\vec{E}$ is not parallel to $\vec{B}$

(d) $\vec{E} \,||\, \vec{B}$ but $\vec{v}$ is not parallel to $\vec{E}$

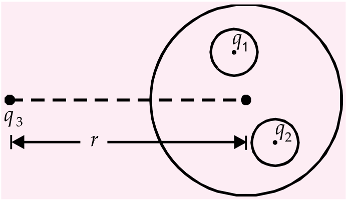
5. A disc of mass 35 kg and radius of gyration 0.75 m is rotating 600 rev/min as shown in figure. What force F must be applied to braking mechanism to stop the disc in 25 seconds? Coefficient of friction between the disc and rod is 0.3.



(a) 91.34 N (b) 39.30 N

(c) 95.83 N (d) 85.00 N

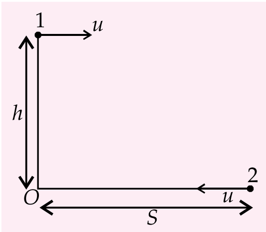
6. A spherical conductor contains two spherical cavities. The total charge on the conductor itself is zero. However, there are the point charges $+q_1$, $+q_2$ at the centre of two cavities, which may be unsymmetrical.



At a considerable distance r away from the centre of the spherical conductor, there is another charge $+q_3$. Forces acting on q_1, q_2 and q_3 are F_1, F_2 and F_3 respectively. Choose the incorrect statement(s).

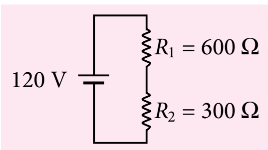
(a) $F_1 < F_2 < F_3$ (b) $F_1 = F_2 < F_3$
(c) $F_1 = F_2 > F_3$ (d) $F_1 > F_2 > F_3$

7. Two particles 1 and 2 are projected with same speed u as shown in figure. Particle 2 is on the ground and moves without friction on the horizontal surface. Particle 1 is initially at a height h from the ground and at a horizontal distance S from particle 2. If a graph is plotted between u and S for the condition of collision of the two then (u on y-axis and S on x-axis)



(a) It will be a parabola passing through origin
(b) It will be a straight line passing through the origin and having a slope of $\sqrt{\frac{g}{8h}}$.
(c) It will be a straight line passing through the origin and having a slope of $\sqrt{\frac{g}{4h}}$.
(d) It will be a parabola not passing through the origin.

8. In the circuit shown in the figure the battery is ideal. A voltmeter of resistance 600 Ω is connected in turn across R_1 and R_2, giving readings V_1 and V_2 respectively. Then



(a) $V_1 = 80$ V (b) $V_1 = 60$ V
(c) $V_2 = 30$ V (d) $V_2 = 40$ V

9. A sound wave of angular frequency ω travels with a speed v in a medium of density ρ and bulk modulus B. Let k be the propagation constant. If P and A are the pressure amplitude and displacement amplitude respectively, then the intensity of sound wave is

(a) $\frac{1}{2}\omega BkA^2$ (b) $\frac{vP^2}{2B}$
(c) $\frac{P^2}{2\rho v}$ (d) $\frac{P^2}{2\sqrt{\rho B}}$

10. Water is flowing smoothly through a closed pipe system. At one point A, the speed of the water is 3.0 m s^{-1} while at another point B, 1.0 m higher, the speed is 4.0 m s^{-1}. The pressure at A is 20 kPa when the water is flowing and 18 kPa when the water flow stops. Then

(a) the pressure at B when water is flowing is 6.7 kPa.
(b) the pressure at B when water is flowing is 8.2 kPa.
(c) the pressure at B when water stops flowing is 10.2 kPa.
(d) the pressure at B when water stops flowing is 8.2 kPa.

SECTION-2

One Integer Value Correct Type

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

11. A pop gun consists of a tube 25 cm long closed at one end by a cork and at the other end by a tightly fitted piston. The piston is pushed slowly in. When the pressure rises to one and half times the atmospheric pressure, the cork is violently blown out. The frequency of the pop caused by its ejection is $170n$ Hz. Find n. (v = 340 m s^{-1})

12. A hydrogen-like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find Z.

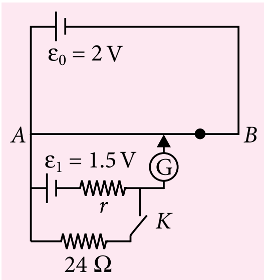
13. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km h^{-1}?

14. A force F = 2.0 N acts on a particle P in the xz-plane. The force F is parallel to x-axis. The particle P is at a distance 3 m and the line joining

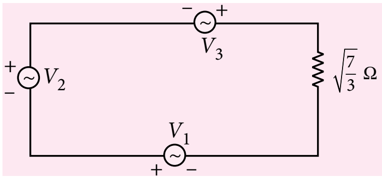
P with the origin makes angle 30° with the x-axis. What is the magnitude of torque on P with respect to origin O (in N m)?

15. A box is kept on a rough horizontal surface. A horizontal force just strong enough to move the box is applied. This force is maintained for 2 seconds, and is then removed. The total distance moved by the box is S meter. Then find the value of $3S$. (Take $\mu_s = 0.20$ and $\mu_k = 0.15$)

16. For the arrangement of the potentiometer shown in the figure, the balance point is obtained at a distance 75 cm from A when the key K is open. The second balance point is obtained at 60 cm from A when the key K is closed. Find the internal resistance (in Ω) of the battery ε_1.



17. Three alternating voltage sources $V_1 = 3 \sin \omega t$ V, $V_2 = 5 \sin (\omega t + \phi_1)$ V and $V_3 = 5 \sin (\omega t - \phi_2)$ V connected across a resistance $R = \sqrt{\frac{7}{3}}\ \Omega$ as shown in the figure (where ϕ_1 and ϕ_2 corresponds to 30° and 127° respectively). Find the peak current (in amp) through the resistor.



18. Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finishing. The mass of S_1 is thrice that of S_2. Both the spheres are heated to the same high temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is $\left(\frac{1}{n}\right)^{\frac{1}{3}}$. Find the value of n.

19. There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially both have equal number of nuclei. After n half lives of A rate of disintegration of both are equal. What is the value of n?

20. A particle of mass m attached to a string of length l is describing circular motion on a smooth plane inclined at an angle α with the horizontal. For the particle to reach the highest point its velocity at the lowest point should exceed : $\sqrt{ngl \sin\alpha}$. Find the value of n.

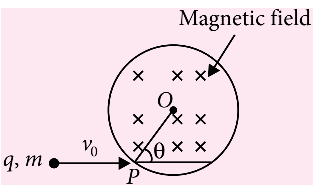
PAPER-2

SECTION-1

Only One Option Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

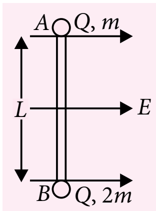
1. A particle of charge q and mass m is projected with a velocity v_0 towards a circular region having uniform magnetic field B perpendicular and into the plane of paper, from point P as shown in figure. R is the radius and O is the centre of the circular region. If the line OP makes an angle θ with the direction of v_0 then the value of v_0 so that particle passes through O is



(a) $\dfrac{qBR}{m\sin\theta}$ (b) $\dfrac{qBR}{2m\sin\theta}$

(c) $\dfrac{2qBR}{m\sin\theta}$ (d) $\dfrac{3qBR}{2m\sin\theta}$

2. Two small balls A and B of positive charge Q each and masses m and $2m$ respectively are connected by a non conducting light rod of length L. This system is released in a uniform electric field of strength E as shown. Just after the release (assume no other force acts on the system)



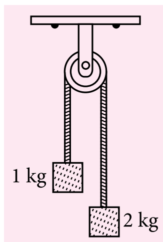
(a) rod has zero angular acceleration

(b) rod has angular acceleration $\dfrac{QE}{2mL}$ in anticlockwise direction.

(c) acceleration of point A is $\dfrac{2QE}{3m}$ towards right

(d) acceleration of point A is $\dfrac{QE}{m}$ towards right

3. Two unequal masses are connected on two sides of a light string passing over a light and smooth pulley as shown in figure. The system is released from rest. The larger mass is stopped for a moment, 1.0 s after the system is set into motion. The time elapsed before the string is tight again is (Take $g = 10$ m s^{-2})



(a) 1/4 s (b) 1/2 s
(c) 2/3 s (d) 1/3 s

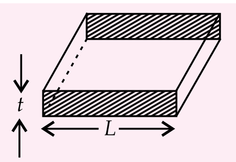
4. A block of mass 1 kg is attached to one end of a spring of force constant $k = 20\ N\ m^{-1}$. The other end of the spring is attached to a fixed rigid support. This spring block system is made to oscillate on a rough horizontal surface ($\mu = 0.04$). Initial displacement of the block from the equilibrium position is $a = 30$ cm. How many times the block passes from the mean position before coming to rest? (Take $g = 10\ m\ s^{-2}$)
(a) 11 (b) 7
(c) 6 (d) 15

5. An object of mass 0.2 kg executes simple harmonic oscillations along the x-axis with a frequency $\frac{25}{\pi}$ Hz. At the position $x = 0.04$ m, the object has kinetic energy 0.5 J and potential energy 0.4 J. The amplitude of oscillation is
(a) 6 cm (b) 4 cm
(c) 9 cm (d) 2 cm

6. In Young's double slit experiment, on introducing a thin sheet of mica of thickness 12×10^{-5} cm in the path of one of the interfering beams, the central fringe is shifted through a distance equal to spacing between the successive bright fringes. The refractive index of mica, if the wavelength of light used is 6×10^{-5} cm will be
(a) 1.6 (b) 1.4
(c) 1.5 (d) 1.55

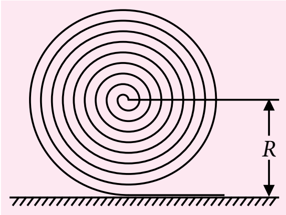
7. A galvanometer of resistance 50 Ω is connected to a battery of 3 V along with a resistance of 2950 Ω in series. A full scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce this deflection to 20 divisions, the resistance to be added in series should be
(a) 6050 Ω (b) 4450 Ω
(c) 5050 Ω (d) 5550 Ω

8. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is



(a) directly proportional to L
(b) directly proportional to t
(c) independent of L
(d) independent of t

9. A reeled carpet of radius R is gently pushed. If it rolls without sliding and unreels, the speed of the centre of mass of the rolling portion when its radius is halved is (Neglect the vertical motion of the centre of mass)



(a) $\sqrt{\frac{7}{3}gR}$ (b) $\sqrt{\frac{14}{3}gR}$
(c) $\sqrt{\frac{14R}{3g}}$ (d) $\sqrt{\frac{7R}{3g}}$

10. A concave mirror has the form of a hemisphere of radius $R = 20$ cm. A thin layer of an unknown transparent liquid is poured into this mirror and it was found that the given optical system with the source in a certain position produces two real images, one of which (formed by direct reflection) coincides with the source and the other is at a distance 8 cm from it. The refractive index μ of the liquid is
(a) $\frac{7}{6}$ (b) $\frac{6}{7}$
(c) $\frac{4}{3}$ (d) $\frac{3}{2}$

SECTION-2

Comprehension Type (Only One Option Correct)

This section contains 3 paragraphs, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (a), (b), (c) and (d).

Paragraph for questions 11 and 12

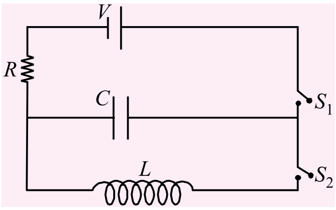
A metal block is placed in a room which is at 10°C for long time. Now it is heated by an electric heater of power 500 W till its temperature becomes 50°C. Its initial rate of rise of temperature is $2.5°C\ s^{-1}$. The heater is switched off and now a heater of 100 W is required to maintain the temperature of the block at 50°C. (Assume Newton's law of cooling to be valid)

11. What is the heat capacity of the block?
(a) $50\ J\ °C^{-1}$ (b) $100\ J\ °C^{-1}$
(c) $150\ J\ °C^{-1}$ (d) $200\ J\ °C^{-1}$

12. What is the rate of cooling of block at 50°C if the 100 W heater is also switched off?
(a) $5°C\ s^{-1}$ (b) $0.5°C\ s^{-1}$
(c) $1°C\ s^{-1}$ (d) $0.1°C\ s^{-1}$

Paragraph for questions 13 and 14

In the given circuit the capacitor (C) may be charged through resistance R by battery V by closing switch S1. Also when S_1 is opened and S_2 is closed the capacitor is connected in series with inductor (L).



13. At the start, the capacitor was uncharged. When switch S_1 is closed and S_2 is kept open, the time constant of this circuit is τ. Which of the following is correct?
(a) After time interval τ, charge on the capacitor $\frac{CV}{2}$ is.
(b) After time interval 2τ, charge on the capacitor of $CV(1 - e^{-2})$
(c) The work done by the voltage source will be half of the heat dissipated when the capacitor is fully charge
(d) After time interval 2τ, charge on the capacitor is $CV(1 - e^{-1})$

14. When the capacitor gets charged completely, S1 is opened and S_2 is closed. Then,
(a) at $t = 0$, energy stored in the circuit is purely in the form of magnetic energy
(b) at any time $t > 0$, current in the circuit is in the same direction
(c) at $t > 0$, there is no exchange of energy between the inductor and capacitor
(d) at any time $t > 0$, instantaneous current in the circuit may be $V\sqrt{\frac{C}{L}}$

Paragraph for questions 15 and 16

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

15. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}$ Hz. Then the moment of inertia of CO moelcule about its center of mass is close to (Take $h = 2\pi \times 10^{-34}$ J s)
(a) 2.76×10^{-46} kg m^2 (b) 1.87×10^{-46} kg m^2
(c) 4.67×10^{-47} kg m^2 (d) 1.17×10^{-47} kg m^2

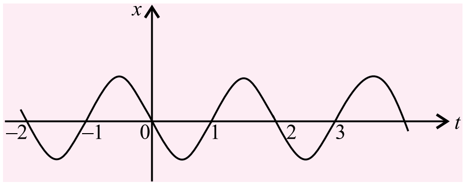
16. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where 1 a.m.u. = $\frac{5}{3}\times 10^{-27}$ kg, is close to
(a) 2.4×10^{-10} m (b) 1.9×10^{-10} m
(c) 1.3×10^{-10} m (d) 4.4×10^{-11} m

SECTION-3

Matching List Type (Only One Option Correct)

This section contains four questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (a), (b), (c) and (d), out of which one is correct.

17. Figure gives the x-t plot of a particle executing one-dimensional simple harmonic motion.



Match the List-I with List-II

List-I Time		List-II Signs of position (x), velocity (v) and acceleration (a)	
P.	At $t = -1.2$ s	1.	$x < 0, v < 0, a > 0$
Q.	At $t = -0.3$ s	2.	$x > 0, v > 0, a < 0$
R.	At $t = 0.3$ s	3.	$x > 0, v < 0, a < 0$
S.	At $t = 1.2$ s	4.	$x < 0, v > 0, a > 0$

Code:
(a) P - 4, Q - 3, R - 1, S - 2
(b) P - 3, Q - 1, R - 2, S - 4
(c) P - 4, Q - 3, R - 2, S - 1
(d) P - 3, Q - 4, R - 1, S - 2

18. In the following, List-I lists some physical quantities and the List-II gives approximate energy values associated with some of them. Choose the appropriate value of energy from List-II for each of the physical quantities in List-I.

List-I		List-II	
P.	Energy of thermal neutrons	1.	0.025 eV
Q.	Energy of X-rays	2.	8 MeV
R.	Binding energy per nucleon	3.	3 eV
S.	Photoelectric threshold of a metal	4.	10 keV

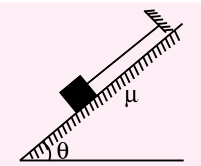
Code:
(a) P - 1, Q - 2, R - 3, S - 4
(b) P - 3, Q - 1, R - 2, S - 4
(c) P - 1, Q - 4, R - 2, S - 3
(d) P - 3, Q - 4, R - 1, S - 2

19. The velocity of an aircraft as seen by the driver of a car is 5 m s^{-1} upwards. A passenger in a train simultaneously sees the car to move southwards with 5 m s^{-1}. The conductor of a bus feels that the train is moving north with a velocity of 10 m s^{-1}. A decoit running towards the bus feels it moving 6 m s^{-1} eastwards. A police jeep chasing the decoit feels him to be moving westwards with 3 m s^{-1}. A person standing on the ground sees the police jeep moving north-west with $15\sqrt{2}$ m s^{-1}.

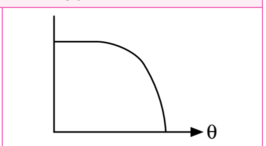
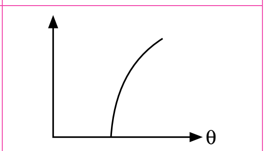
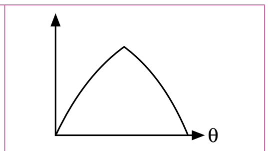
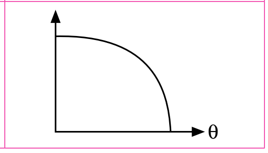
	List-I		List-II
P.	Velocity of the aircraft as seen by the conductor	1.	$-3\hat{i}-5\hat{j}-5\hat{k}$
Q.	Velocity of the conductor as seen by the police	2.	$5\hat{j}+5\hat{k}$
R.	Velocity of the aircraft as seen by the person on ground	3.	$3\hat{i}$
S.	Velocity of police as seen by pilot of aircraft	4.	$-12\hat{i}+20\hat{j}+5\hat{k}$

Code:
(a) P - 1, Q - 2, R - 3, S - 4
(b) P - 2, Q - 3, R - 4, S - 1
(c) P - 1, Q - 4, R - 2, S - 3
(d) P - 2, Q - 3, R - 1, S - 4

20. A block of mass m is put on a rough inclined plane of inclination θ, and is tied with a light thread shown. Inclination θ is increased gradually from θ = 0° to θ = 90°.



Match the column according to corresponding curves.

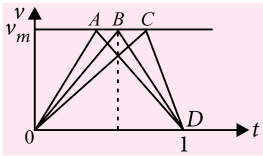
	List-I		List-II
P.	Tension in the thread versus θ	1.	
Q.	Normal reaction between the block and the incline versus θ	2.	
R.	Friction force between the block and the incline versus θ	3.	
S.	Net interaction force between the block and the incline versus θ	4.	

Code:
(a) P - 2, Q - 4, R - 3, S - 1
(b) P - 3, Q - 1, R - 2, S - 4
(c) P - 2, Q - 3, R - 4, S - 1
(d) P - 3, Q - 4, R - 1, S - 2

SOLUTIONS

Paper-1

1. **(a, c, d) :** The body is at rest at $t = 0$ and $t = 1$.
Initially α is positive so that the body acquires some velocity. Then α should be negative so that the body comes to rest. Hence α cannot remain positive for all time in the interval $0 \le t \le 1$
The journey is depicted in the following v-t graph.
Total time of journey = 1 sec
Total displacement = 1 m
= Area under (v-t) graph



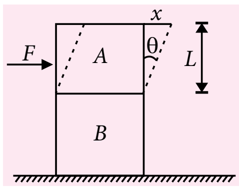
$$v_{max} = \frac{2s}{t} = \frac{2\times1}{1} = 2 \text{ m s}^{-1}$$

For path OB, acceleration (α)

$$= \frac{\text{Change in velocity}}{\text{time}} = \frac{2}{1/2} = 4 \text{ m s}^{-2}$$

For path BD, retardation = – 4 m s^{-2}
For path OA, α (acceleration) > 4 m s^{-2}
For path AD, α (retardation) < – 4 m s^{-2}
For path OC, acceleration α < 4 m s^{-2}
For path CD, retardation α > – 4 m s^{-2}
Hence $|\alpha| \ge 4$ at some point or points in its path.
Hence (a), (c) and (d) are correct options.

2. **(d) :** The cubical block A is placed above cubical block B. They have same dimensions.
The lower face of B is held rigidly on a horizontal surface. A force F is applied to the upper cube A at right angles to one of the side faces. The block A executes SHM when the force is withdrawn. The lower block gets distorted.



∴ Modulus of rigidity,

$$\eta = \frac{-F}{A\theta}$$

$$= -\frac{F}{(L^2)\left(\frac{x}{L}\right)} = -\frac{F}{Lx}$$

$\Rightarrow$ Restoring force $F = -\eta L x$

$\Rightarrow$ Acceleration $= \frac{F}{M} = -\frac{\eta L x}{M}$

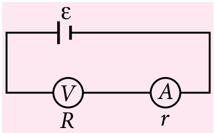
or Acceleration $= -\omega^2 x$ where $\omega^2 = \frac{\eta L}{M}$.

Obviously the motion is simple harmonic.

$\therefore \ \omega^2 = \frac{\eta L}{M}$ or $\left(\frac{2\pi}{T}\right)^2 = \frac{\eta L}{M}$ or $T = 2\pi\sqrt{\frac{M}{\eta L}}$.

3. (b, d): In first case, $V = \frac{\varepsilon \times R}{R+r}$ and $A = \frac{\varepsilon}{R+r}$

In second case, $V' = \frac{\varepsilon \times R}{R + \frac{r}{2}}$



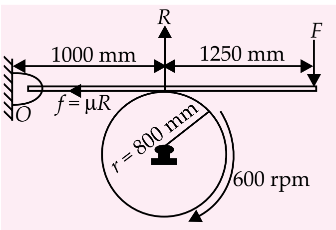
$$A' = \frac{1}{2}\left(\frac{\varepsilon}{R + \frac{r}{2}}\right) = \frac{\varepsilon}{2R+r} > \frac{\varepsilon}{2(R+r)}$$

Clearly V increases and A becomes slightly more than half of its initial value.

4. (a, b) : If electric field is parallel to magnetic field, the charged particle moving parallel to $\vec{E}$ experience a force in the direction of $\vec{E}$; due to $\vec{B}$ there will not be any force. Hence, no deflection.

The particle may go undeflected in the case when forces due to electric field and magnetic field balance each other.

5. (a) :



Taking moment about O,

$F \times (1000 + 1250) - R \times 1000 + \mu R \times 0 = 0$

$\therefore \ R = \frac{2250}{1000} F = 2.25\,F$

$\therefore$ Frictional force, $f = \mu R = 0.3 \times 2.25F = 0.675F$

Braking torque, $\tau = f \times r = 0.675F \times \frac{800}{1000}$ N m

$\tau = 0.54F$

Also, $\tau = I\alpha$

$\therefore \ I\alpha = 0.54F$ or $\alpha = \frac{0.54F}{I} = \frac{0.54F}{mk^2}$

$\alpha = \frac{0.54F}{35 \times (0.75)^2} = 0.0275F$

Now, $t = \frac{\omega - \omega_0}{\alpha} \Rightarrow 25 = \frac{2\pi \times \frac{600}{60}}{0.0275F}$

Solving, we get, $F = 91.34$ N

6. (a, c, d) **7. (b)**

8. (b, c) : Equivalent resistance of R_1 (= 600 Ω) and voltmeter (= 600 Ω) in parallel is 300 Ω.

Total resistance in the circuit = 300 Ω + 300 Ω

= 600 Ω

Since current, $I = \frac{120\text{ V}}{600\ \Omega} = 0.2$ A,

Potential difference (V_1) across R_1 and voltmeter = 0.2 A × 300 Ω = 60 V

Equivalent resistance of R_2 (= 300 Ω) and voltmeter (= 600 Ω) in parallel is 200 Ω.

Total resistance in the circuit = 200 Ω + 600 Ω

= 800 Ω

Since current, $I' = \frac{120\text{ V}}{800\ \Omega} = 0.15$ A,

Potential difference (V_2) across R_2 and voltmeter = (0.15 A) (200 Ω) = 30 V

9. (a, b, c, d) : Intensity, by definition, is the energy flowing per unit area per unit time.

The intensity is related to the displacement amplitude A of the sound wave by

$$I = \frac{1}{2}\rho v \omega^2 A^2$$

The displacement amplitude is given by $A = \frac{P}{Bk}$, where $k\left(=\frac{\omega}{v}\right)$ is the propagation constant.

The speed is given by $v = \sqrt{\frac{B}{\rho}}$.

Use these relations to get the required expressions.

10. (a, d) : Let P_1, h_1, v_1 and P_2, h_2, v_2 represent the pressures, heights and velocities of flow at the two points A and B respectively. According to the Bernoulli's theorem

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \quad \text{...(i)}$$

Putting v_1 = 3.0 m s^{-1}, v_2 = 4.0 m s^{-1}, $(h_2 - h_1)$ = 1 m, P_1 = 20 kPa

we get,

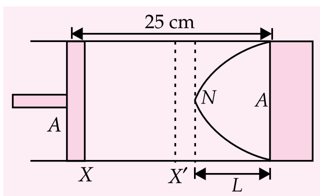
$$P_2 = 20 + \left[10^3 \times 9.8\,(-1) + \frac{10^3}{2}[9 - 16]\right] \times 10^{-3}$$

= 20 − 9.8 − 3.5 = 6.7 kPa

Also when the flow stops, $v_1 = v_2 = 0$ and then from (i),

P_2 = 18 − 9.8 = 8.2 kPa

11. (3) :



Let A be area of cross-section.
In isothermal process,

$$P_1V_1 = P_2V_2$$

$$P \times 25 \times A = \frac{3}{2} \times P \times L \times A \Rightarrow L = \left(\frac{50}{3}\right) \text{cm}$$

Now, after the ejection of cork, for oscillating air node will be at piston while antinode will be at the open end and as minimum distance between node and antinode is ($\lambda/4$).

So, $\frac{\lambda}{4} = L = \frac{50}{3}$ cm $\Rightarrow \lambda = \frac{2}{3}$ m

and hence, $\upsilon = \frac{v}{\lambda} = \frac{340 \times 3}{2} = 510$ Hz $= 170 \times 3$ Hz

Hence, $n = 3$

12. (4) : The energy of n^{th} orbit of hydrogen-like atom is

$$E_n = -\frac{13.6Z^2}{n^2}$$

Maximum energy corresponds to transition $2n \to 1$

$$\therefore \quad 204 = 13.6Z^2\left(\frac{1}{1} - \frac{1}{(2n)^2}\right) \quad \text{...(i)}$$

Also for transition $2n \to n$

$$40.8 = 13.6Z^2\left(\frac{1}{n^2} - \frac{1}{4n^2}\right) \text{ or } 40.8 = 13.6Z^2\left(\frac{3}{4n^2}\right)$$

or $\quad 40.8 = 40.8\frac{Z^2}{4n^2}$ or $4n^2 = Z^2$

or $\quad 2n = Z$ ...(ii)

From (i) and (ii), we get

$$204 = 13.6Z^2\left(1 - \frac{1}{Z^2}\right) = 13.6Z^2 - 13.6$$

$13.6\,Z^2 = 204 + 13.6 = 217.6$

$$Z^2 = \frac{217.6}{13.6} = 16 \text{ or } Z = 4$$

13. (3) : Focal length of a convex mirror,

$$f = \frac{R}{2} = \frac{20}{2} \text{ m} = 10 \text{ m}$$

$$v_1 = +\frac{25}{3} \text{ m}, \ f = +10 \text{ m}$$

Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\therefore \quad \frac{1}{(25/3)} + \frac{1}{u_1} = \frac{1}{10} \text{ or } \frac{1}{u_1} = \frac{1}{10} - \frac{3}{25}$$

or $u_1 = -50$ m

$$v_2 = +\frac{50}{7} \text{ m}, \ f = +10 \text{ m}$$

$$\therefore \quad \frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f}$$

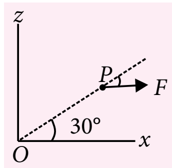
$$\frac{1}{(50/7)} + \frac{1}{u_2} = \frac{1}{10} \text{ or } \frac{1}{u_2} = \frac{1}{10} - \frac{7}{50}$$

or $u_2 = -25$ m

Speed of the object $= \frac{25}{30}$ m s^{-1}

$$= \frac{25}{30} \times \frac{18}{5} \text{ km h}^{-1} = 3 \text{ km h}^{-1}$$

14. (3) : Torque, $\vec{\tau} = \vec{r} \times \vec{F}$



In magnitude, $\tau = rF\sin\theta$
where θ is the angle between $\vec{r}$ and $\vec{F}$.
Here, $F = 2.0$ N, $r = 3$ m, $\theta = 30°$
$\therefore$ The magnitude of torque on P with respect to origin O is

$$\tau = (3 \text{ m})(2.0 \text{ N}) \sin 30°$$

$$= (3 \text{ m})(2.0 \text{ N})\left(\frac{1}{2}\right) = 3 \text{ N m}$$

15. (4) : Applied force $= \mu_s N = \mu_s mg$, when the block is moving, friction acting on it is $\mu_k mg$

$$\therefore \quad a = \frac{\mu_s mg - \mu_k mg}{m} = (\mu_s - \mu_k)g = 0.05g = 0.5 \text{ m s}^{-2}$$

When the applied force is removed, retardation $= \mu_k\, g = 1.5$ m s^{-2}

Distance covered in 2 s, $d_1 = \frac{1}{2} \times 0.5 \times 2^2 = 1$ m;

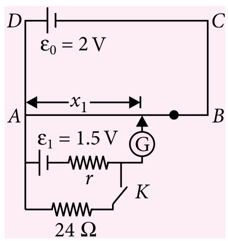
velocity after 2 s $= 0.5 \times 2 = 1$ m s^{-1}

Distance covered during retardation;

$$0^2 = 1^2 - 2(1.5)\, d_2 \Rightarrow d_2 = \frac{1}{3}$$

$$\therefore \quad \text{Total distance } S = 1 + \frac{1}{3} = \frac{4}{3}; \ 3\,S = 4 \text{ m}$$

16. (6) : Let λ is resistance per unit length of wire AB.



When K is opened.

$I(\lambda x_1) = \varepsilon_1$ ... (i)

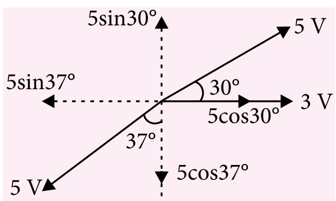
K is closed, $I\lambda x_2 = \varepsilon_1 - ir$ ... (ii)

$$i = \frac{\varepsilon_1}{R + r} \quad \text{... (iii)}$$

From eqns (i), (ii) and (iii), we get

$$r = \left(\frac{x_1}{x_2} - 1\right)R = \left(\frac{0.75}{0.60} - 1\right)24; \ r = 6\ \Omega$$

17. (3) : $V_1 = 3\sin\omega t$; $V_2 = 5\sin(\omega t + \phi_1)$; $V_3 = 5\sin(\omega t - \phi_2)$



$$V_{max} = \sqrt{(3 + 5\cos 30° - 5\sin 37°)^2 + (5\sin 30° - 5\cos 37°)^2}$$

$$= \sqrt{\left(\frac{5\sqrt{3}}{2}\right)^2 + (1.5)^2} = \sqrt{21}\ \text{V}$$

$$I_{max} = \frac{V_{max}}{R} = \frac{\sqrt{21}}{\sqrt{7/3}} = \sqrt{\frac{21\times 3}{7}} = 3\ \text{A}$$

18. (3) : The rate at which energy leaves the object is

$$\frac{\Delta Q}{\Delta t} = e\sigma A T^4$$

Since, $\Delta Q = mC\Delta T$, we get

$$\frac{\Delta T}{\Delta t} = \frac{e\sigma A T^4}{mC}$$

Also, since $m = \frac{4}{3}\pi r^3 \rho$ for a sphere, we get

$$A = 4\pi r^2 = 4\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}$$

Hence, $$\frac{\Delta T}{\Delta t} = \frac{e\sigma T^4}{mC}\left[4\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}\right] = K\left(\frac{1}{m}\right)^{1/3}$$

For the given two bodies

$$\frac{(\Delta T/\Delta t)_1}{(\Delta T/\Delta t)_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

19. (1): Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is λN_0 and that of B is $2\lambda N_0$.

After one half-life of A, rate of disintegration of A will become $\frac{\lambda N_0}{2}$ and that of B would also be $\frac{\lambda N_0}{2}$ (half-life of $B = \frac{1}{2}$ half-life of A)

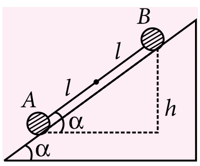
So, after one half-life of A or two half-lives of B.

$$\left(-\frac{dN}{dt}\right)_A = \left(-\frac{dN}{dt}\right)_B$$

$\therefore\ n = 1$

20. (5) : $h = 2\,l \sin\alpha$

A is the lowest point and B is the highest point. At B, in critical case tension is zero. Let velocity of particle at B at this instant be v_B. Then



$$mg\sin\alpha = \frac{mv_B^2}{l}$$

or $\quad v_B^2 = gl\sin\alpha$

Now $\quad v_A^2 = v_B^2 + 2gh$

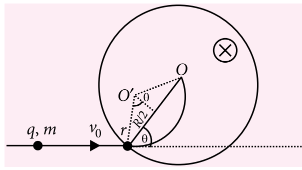
$= (gl\sin\alpha) + 2g\,(2l\sin\alpha)$

$\therefore\quad v_A = \sqrt{5gl\sin\alpha}$

Paper-2

1. (b) : Let r be the radius of circular path. If the particle passes through O, then from the given figure,

$$\sin\theta = \frac{R/2}{r} = \frac{R}{2r} \quad ...(i)$$



As $r = \frac{mv_0}{qB}$

$$\therefore\ v_0 = \frac{qBr}{m}$$

$$= \frac{qB(R/2\sin\theta)}{m} \quad \text{(Using (i))}$$

$$= \frac{qBR}{2m\sin\theta}$$

2. (d) : Resultant force on arrangement is $2QE$, thus acceleration of centre of mass is given by

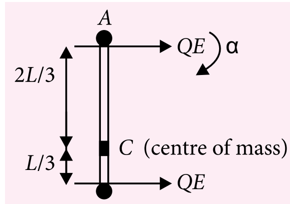
$$a_C = \frac{2QE}{3m} \quad \text{(towards right)}$$

As per conservation of angular momentum

$$QE\left(\frac{2L}{3}\right) - QE\left(\frac{L}{3}\right) = \left[m\times\frac{4L^2}{9} + 2m\times\frac{L^2}{9}\right]\alpha$$

$$\alpha = \frac{QE}{2mL} \quad \text{(clockwise)}$$

From constraint equation, we get



$$a_A = a_C + \alpha\left(\frac{2L}{3}\right)$$

$$= \frac{2QE}{3m} + \frac{QE}{2mL}\left(\frac{2L}{3}\right)$$

$$= \frac{3QE}{3m} = \frac{QE}{m} \quad \text{(towards right)}$$

3. (d) : Net pulling force = $2\,g - 1\,g = 10$ N

Mass being pulled = 2 + 1 = 3 kg

$\therefore$ Acceleration of the system is $a = \frac{10}{3}$ m s^{-2}

$\therefore$ Velocity of both the blocks at $t = 1$ s will be

$$v_0 = at = \left(\frac{10}{3}\right)(1) = \frac{10}{3}\ \text{m s}^{-1}$$

Now, at this moment velocity of 2 kg block becomes zero, while that of 1 kg block is $\frac{10}{3}$ m s^{-1} upwards. Hence, string becomes tight again when displacement of 1 kg block equals displacement of 2 kg block.

i.e., $v_0 t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

or $t = \frac{v_0}{g} = \frac{10/3}{10} = \frac{1}{3}$ s

4. (b) : Let the initial amplitude a decreases to a_1 to the other side, *i.e.*, after the first sweep ;
then decrease in elastic potential energy
= work done against friction

or $\frac{1}{2}ka^2 - \frac{1}{2}ka_1^2 = \mu mg(a + a_1)$

or $\frac{1}{2}k(a + a_1)(a - a_1) = \mu mg\ (a + a_1)$

or $a - a_1 = \frac{2\mu mg}{k}$

Similarly, $a_1 - a_2 = \frac{2\mu mg}{k}$

....

$a_{n-1} - a_n = \frac{2\mu mg}{k}$

Adding all the equations

$$a - a_n = \frac{2n\mu mg}{k} \quad \text{...(i)}$$

The block stops when

$\mu mg = ka_n$ or $a_n = \frac{\mu mg}{k}$.

Substituting in eqn. (i), we get

$$(2n+1)\left(\frac{\mu mg}{k}\right) = a$$

or $(2n + 1) = \frac{ka}{\mu mg} = \frac{(20)(0.3)}{(0.04)(1)(10)} = 15$

$\therefore$ $n = 7$

5. (a) : As $\omega = 2\pi\upsilon = \sqrt{\frac{k}{m}}$

$\therefore$ $k = (2\pi\upsilon)^2 m$

Total energy of oscillation is
$E = 0.5 + 0.4 = 0.9$ J

$\therefore$ $0.9 = \frac{1}{2}kA^2$

or $A = \sqrt{\frac{1.8}{k}} = \sqrt{\frac{1.8}{(2\pi\upsilon)^2 m}}$

$$= \frac{1}{2\pi\left(\frac{25}{\pi}\right)}\sqrt{\frac{1.8}{0.2}} = \frac{3}{50}\text{ m} = 6 \text{ cm}$$

6. (c)

7. (b) : Total initial resistance
$= G + R = 50\ \Omega + 2950\ \Omega = 3000\ \Omega$

Current, $I = \frac{3\text{ V}}{3000\ \Omega} = 1\times10^{-3}$ A $= 1$ mA

If the deflection has to be reduced to 20 divisions, then current

$$I' = \frac{1\text{ mA}}{30}\times 20 = \frac{2}{3}\text{ mA}$$

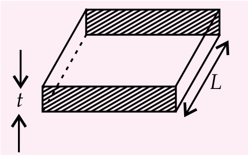
Let x be the effective resistance of the circuit, then

$$3\text{ V} = 3000\ \Omega\times 1\text{ mA} = x\ \Omega\times\frac{2}{3}\text{ mA}$$

or $x = 3000\times 1\times\frac{3}{2} = 4500\ \Omega$

$\therefore$ Resistance to be added = (4500 Ω – 50 Ω) = 4450 Ω

8. (c) : As $R = \frac{\rho l}{A}$



Here, $l = L$, $A = Lt$

$\therefore$ $R = \frac{\rho L}{Lt} = \frac{\rho}{t}$

Hence, the resistance between two opposite faces, shown by the shaded areas in the figure is independent of L.

9. (b) **10. (c)**

11. (d) : $P_{\text{heater}} = P_{\text{given to block}}$

$\therefore$ $500 = C\frac{dT}{dt}$; $500 = 2.5C$

or $C = 200$ J °C^{-1}

12. (b) : Power loss to surroundings = 100 W;

$C\frac{dT}{dt} = -100$

$\frac{dT}{dt} = -\frac{100}{200} = -0.5$°C s^{-1}

13. (b) **14. (d)** **15. (b)**

16. (c) : The moment of inertia of CO molecule is

$I = \mu r^2$ ...(i)

where,
μ = reduced mass of the CO molecule
r = distance between C and O or bond length

The reduced mass μ of the CO molecule is

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \left[\frac{(12)(16)}{12+16}\right]\times\frac{5}{3}\times10^{-27}\text{ kg}$$

From equation (i), we get

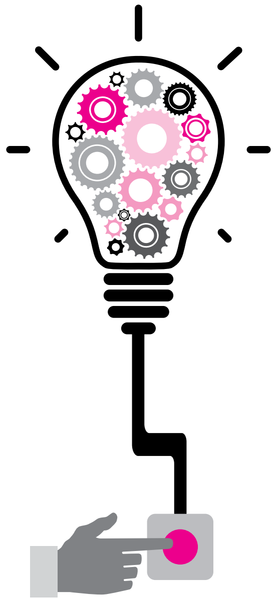
$$r^2 = \frac{I}{\mu} = \frac{1.87\times10^{-46}}{\left[\frac{12\times16}{28}\times\frac{5}{3}\times10^{-27}\right]}$$

or $r = 1.3\times10^{-10}$ m

17. (a) **18. (c)** **19. (b)** **20. (a)**

❖❖

THOUGHT PROVOKING PROBLEMS

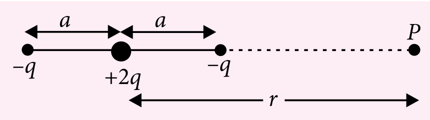


By : Prof. Rajinder Singh Randhawa*

ELECTROSTATICS

1. A small bead of mass m, charge $-q$ is constrained to move along a frictionless wire. A positive charge Q lies at a distance L from the wire. Show that if the bead is displaced a distance x, where $x << L$, and released, it will exhibit simple harmonic motion. Obtain an expression for the time period of simple harmonic motion.

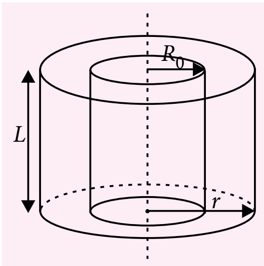
2. Three charges $-q$, $+2q$ and $-q$ are arranged on a line as shown in figure. Calculate the field at a distance $r > a$ on the line.



3. A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution.

4. A circular ring of radius R with uniform positive charge density λ per unit length is located in the Y-Z plane with its centre at the origin O. A particle of mass m and positive charge $+q$ is projected from the point $P(R\sqrt{3}, 0, 0)$ on the positive X-axis directly towards O, with initial velocity v. Find the smallest (non-zero) value of the speed v such that the particle does not return to P.

5. A straight infinitely long cylinder of radius R_0 is uniformly charged having surface charge density σ. The cylinder serves as a source of electrons, with the velocity vector of emitted electrons perpendicular to its surface. What must be electron velocity to ensure that the electron can move away from the axis of the cylinder to a distance greater than r.



6. A spherical shell of radius R_1 with a uniform charge q has a point charge q_0 at its centre. Find the work performed by the electric forces during the shell expansion from radius R_1 to radius R_2.

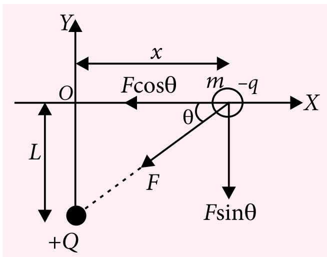
SOLUTIONS

1. From figure, applying Newton's 2nd law,

$$-F\cos\theta\,\hat{i} = m\vec{a}$$

$$-\frac{Qq}{4\pi\varepsilon_0(x^2+L^2)}\frac{x}{\sqrt{x^2+L^2}}\hat{i} = m\vec{a}$$

$$\therefore\ \vec{a} = -\frac{Qq}{4\pi\varepsilon_0 m}\frac{x}{(x^2+L^2)^{3/2}}\hat{i}$$



For small linear displacement, $x << L$, so x^2 is neglected.

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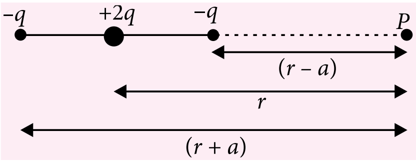
$\therefore \quad \vec{a} = -\dfrac{Qq}{4\pi\varepsilon_0 mL^3} x\hat{i}$

which is equation of SHM with

$\omega = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 mL^3}}$

As $\omega = \dfrac{2\pi}{T} \quad \therefore \; T = 2\pi\sqrt{\dfrac{4\pi\varepsilon_0 mL^3}{Qq}}$

2. 

From figure,

$$\vec{E}_1 = -\frac{q}{4\pi\varepsilon_0 (r-a)^2}$$

$$\vec{E}_2 = \frac{2q}{4\pi\varepsilon_0 r^2} \text{ and } \vec{E}_3 = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}$$

$$\therefore \quad \vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{(r-a)^2} + \frac{2}{r^2} - \frac{1}{(r+a)^2}\right]$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{1-\left(\frac{a}{r}\right)\right\}^{-2} + 2 - \left\{1+\left(\frac{a}{r}\right)\right\}^{-2}\right]$$

If $r > a$, we can use binomial approximation,

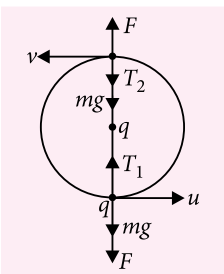
$(1 + x)^n \approx 1 + nx + \dfrac{(n)(n-1)}{2!}x^2$ for $x < 1$

$$\therefore \; \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left(1+\frac{2a}{r}+\frac{3a^2}{r^2}\right)+2-\left(1-\frac{2a}{r}+\frac{3a^2}{r^2}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\left[-1-\frac{2a}{r}-\frac{3a^2}{r^2}+2-1+\frac{2a}{r}-\frac{3a^2}{r^2}\right]$$

$$= -\frac{6a^2 q}{4\pi\varepsilon_0 r^4}$$

3. If the ball has to just complete the circle then the tension must vanish at the topmost point, *i.e.* $T_2 = 0$.



From Newton's 2nd law

$$T_2 + mg - \frac{q^2}{4\pi\varepsilon_0 l^2} = \frac{mv^2}{l} \quad \text{...(i)}$$

As $T_2 = 0$, then

$$mg - \frac{q^2}{4\pi\varepsilon_0 l^2} = \frac{mv^2}{l} \quad \text{...(ii)}$$

Using conservation of energy

Energy at topmost point = Energy at lowest point

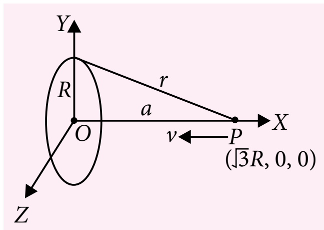
$$\frac{1}{2}mv^2 + mg(2l) = \frac{1}{2}mu^2$$

$$v^2 = u^2 - 4gl \quad \text{...(iii)}$$

Put (iii) in (ii), we get

$$u = \sqrt{5gl - \frac{q^2}{4\pi\varepsilon_0 ml}}$$

Put the given values, we get $u = 5.8 \text{ m s}^{-1}$

4. 

From figure,

Potential at the centre of ring,

$$V = \frac{\lambda R}{2\varepsilon_0\sqrt{a^2 + R^2}}$$

$$V = \frac{\lambda R}{2\varepsilon_0\sqrt{(\sqrt{3}R)^2 + R^2}} = \frac{\lambda}{4\varepsilon_0}$$

$\therefore$ Potential energy at $P = \dfrac{q\lambda}{4\varepsilon_0}$

$\therefore$ Total energy at $P = \dfrac{\lambda q}{4\varepsilon_0} + \dfrac{1}{2}mv^2$

The potential energy at centre $= \dfrac{\lambda q}{2\varepsilon_0}$

The particle will not return to P, when

$$\frac{\lambda q}{4\varepsilon_0} + \frac{1}{2}mv^2 = \frac{\lambda q}{2\varepsilon_0}$$

Solving, we get $v = \sqrt{\dfrac{\lambda q}{2\varepsilon_0 m}}$

5. According to Gauss's theorem,

$$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0} \Rightarrow E\,(2\pi rL) = \frac{\sigma(2\pi R_0 L)}{\varepsilon_0}$$

$$\therefore \quad E = \frac{\sigma R_0}{\varepsilon_0 r} \quad \text{...(i)}$$

Applying Newton's 2nd law,

$$m_e \frac{d^2 r}{dt^2} = -e\frac{R_0 \sigma}{\varepsilon_0 r} \quad \text{...(ii)}$$

Using energy conservation

$$\frac{1}{2} m_e v_0^2 - eV_0 = -eV \quad \text{...(iii)}$$

where V_0 is the potential of the cylinder and V is the potential at a distance r from the cylinder's axis.

Since $E = -\frac{dV}{dr}$, $\therefore \frac{R_0 \sigma}{\varepsilon_0 r} = -\frac{dV}{dr}$

Integrating, we get

$$V = -\frac{R_0 \sigma}{\varepsilon_0} \ln r + C \quad \text{...(iv)}$$

where C is constant of integration.

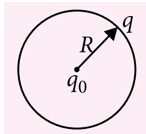
$$\text{Also, } V_0 = -\frac{R_0 \sigma}{\varepsilon_0} \ln R_0 + C \quad \text{...(v)}$$

From eqns. (iii), (iv) and (v), we get

$$v_0 = \sqrt{\frac{2eR_0 \sigma \ln (r/R_0)}{\varepsilon_0 m_e}}$$

6. The electrical potential energy of the system is

$$U = \frac{q^2}{8\pi\varepsilon_0 R} + \frac{qq_0}{4\pi\varepsilon_0 R}$$



Initial potential energy of the system is

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1} + \frac{qq_0}{4\pi\varepsilon_0 R_1}$$

Final potential energy of the system is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2} + \frac{qq_0}{4\pi\varepsilon_0 R_2}$$

From work-energy theorem,

$W = -\Delta U = -(U_f - U_i)$

$$W = U_i - U_f = \frac{q\left[q_0 + \frac{q}{2}\right]}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

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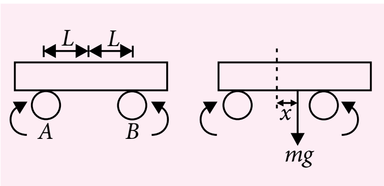
JEE WORK OUTS

Vibration and SHM

1. A uniform bar with mass m lies symmetrically across two rapidly rotating fixed rollers, A and B with distance L = 2.0 cm between the bar's centre of mass and each roller. The rollers , whose directions of rotation are shown in figures slip against the bar with coefficient of kinetic friction μ_k = 0.40. Suppose the bar is displaced horizontally by a distance x as shown in figure and then released. What is the angular frequency ω of the resulting horizontal simple harmonic motion of the bar?

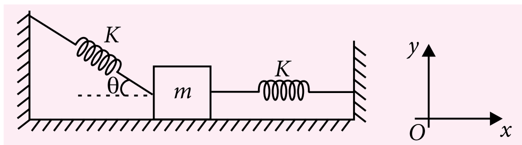


2. A particle of mass m is located in a unidimensional potential field where the potential energy of the particle depends on the coordinates as $U(x) = U_0(1 - \cos ax)$, where U_0 and a are constants. Find the period of small oscillations that the particle performs about the equilibrium position.

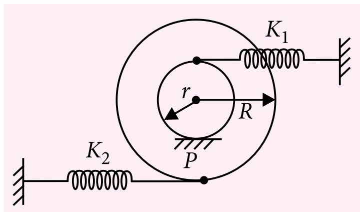
3. The speed v of a particle moving along x-axis is given by $v^2 = 8bx - x^2 - 12b^2$ where b is a constant. Find amplitude of oscillation.

4. A pendulum has a string of length 99.39 cm. How much length of the pendulum must be shortened to keep the current time of the pendulum if it loses 4 s a day.

5. A block of mass m placed on a smooth horizontal floor is connected with two light springs each of stiffness K. Find the time period of small horizontal oscillation of the block along x-axis.



6. A stepped pulley having mass m, radius R and radius of gyration k is connected with two ideal springs of stiffness K_1 and K_2 as shown in figure. If the pulley rolls without sliding, find the angular frequency of its oscillation.

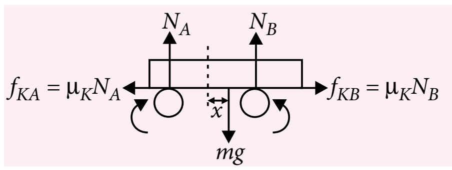


SOLUTIONS

1. Consider horizontal forces

$$\mu_k N_A - \mu_k N_B = ma$$

or $$a = \frac{\mu_k N_A - \mu_k N_B}{m} \quad \ldots(i)$$



Taking torque about an axis perpendicular to the plane and through the contact point between

Aditya Sharma, Prime Classes Pvt Ltd, Durgakund, Varanasi

bar and roller A. The bar experiences no angular acceleration about that axis.

$$N_A(0) + N_B(2L) - mg(L + x) + f_{KA}(0) + f_{KB}(0) = 0 \quad \ldots\text{(ii)}$$

By balancing vertical forces, we have

$$N_A + N_B - mg = 0 \quad \ldots\text{(iii)}$$

Solving eqns. (i), (ii) and (iii)

$$N_B = \frac{mg(L+X)}{2L}, \; N_A = \frac{mg(L-X)}{2L}$$

$$a = -\frac{\mu_k g}{L} X$$

Comparing it with $a = -\omega^2 X$

we get, $\omega = \sqrt{\frac{\mu_k g}{L}} = \sqrt{\frac{0.40 \times 10}{2 \times 10^{-2}}} \simeq 14 \text{ rad s}^{-1}$

2. $U = U_0(1 - \cos ax)$

This is zero when $\cos ax = 1$ or $ax = 0$ or $x = 0$

$\therefore$ $x = 0$ is mean position of system.

At extreme end, potential energy becomes maximum, *i.e.*, $2U_0$

$\therefore$ $2U_0 = U_0(1 - \cos Aa)$ where A is amplitude.

$2 = 1 - \cos Aa$

Since A is small, $\cos aA = 1 - \frac{(Aa)^2}{2}$

$$\therefore \; 1 - \left(1 - \frac{(Aa)^2}{2}\right) = 2$$

or $A = 2/a$

As maximum kinetic energy $= 2U_0$

$$\therefore \; 2U_0 = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m\omega^2 \frac{4}{a^2}$$

$$\text{or} \; \omega^2 = \frac{U_0 a^2}{m} \; \text{or} \; \omega = a\sqrt{\frac{U_0}{m}} \; \Rightarrow T = \frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$$

3. At extreme position v becomes zero, so

$8bx - x^2 - 12b^2 = 0$

or $x^2 - 8bx + 12b^2 = 0$

$(x - 6b)(x - 2b) = 0$

$\therefore$ $x = 2b$ and $6b$

It means particle moves along x–axis from $x = 2b$ to $6b$,

$\therefore$ $2A = 6b - 2b = 4b \;\Rightarrow\; A = 2b$

4. Time lost by pendulum per day = 4 s

$dn = -4$ s

Number of seconds in a day, $n = 24 \times 3600$ s

Let dl be the length by which the pendulum should be shortened to keep the correct time

$$\frac{dn}{n} = -\frac{dl}{2l} \;\Rightarrow\; \frac{-4}{24 \times 3600} = \frac{-dl}{2 \times 99.39}$$

$\Rightarrow$ $dl = 0.00920$ cm

5. When the block is pushed by x along x-axis, right spring will be compressed by x while left spring will be stretched by $x\cos\theta$.

Net restoring force along x-axis,

$F_{net} = Kx + K\,x\cos^2\theta = xK(1 + \cos^2\theta)$

$\Rightarrow$ $K_{eq} = K(1 + \cos^2\theta)$

$$\therefore \; T = 2\pi\sqrt{\frac{m}{K(1+\cos^2\theta)}}$$

6. Torque about contact point P,

$\tau_P = -K_1 x_1 2r - K_2 x_2 (R - r) = I_P\alpha$

Now, $x_1 = 2r\theta$, $x_2 = (R - r)\theta$

$\therefore$ $I_P\alpha = -4K_1 r^2\theta - K_2(R - r)^2\theta$ ...(i)

Also, $I_P = mk^2 + mr^2$ ...(ii)

From eqns. (i) and (ii)

$$\alpha = -\frac{(4K_1 r^2 + K_2(R-r)^2)}{m(k^2 + r^2)}\theta$$

Comparing it with $\alpha = -\omega^2\theta$,we get

$$\omega = \sqrt{\frac{4K_1 r^2 + K_2(R-r)^2}{m(k^2 + r^2)}}$$

SOLUTION OF APRIL 2015 CROSSWORD

1G			2B									3W		4A				5P	I	T	6C	H	
A		7P	E	R	M	E	A	N	C	E		I		N							H		
M			C			8T	E	N	S	I	O	N		I			9M				A		
M			Q						10N			D		S		11H	A	R	M	O	N	I	C
12A	C	C	U	R	A	C	Y		E			V		O			S				N		
N			E						U			A		T		13I	S	O	T	H	E	R	M
G			14R	E	G	E	15L	A	T	I	O	N		R			D				L		
S			E				A		R			E		O			E			16M			
17T	E	S	L	A			M		O					P			F			O		18S	
R							I		N			19F	R	I	N	G	E	S		D		O	
20O	R	B	I	T			N		S					C			C			E		N	
M					21F	R	A	C	T	U	R	22E	P	O	I	N	T			R		O	
							R		A			L								A		G	
				23T			F		R		24V	E	N	T	U	R	I	M	E	T	E	R	
	25E			R			L					C								O		A	
	U		26S	I	P	H	O	N		27B	E	T	A	D	E	C	A	Y		R		P	
	T			G			W					R										H	
	E			G					28A	B	S	O	L	U	T	E	Z	E	R	O		Y	
	C			E								M											
	T		29T	R	U	T	H	T	A	B	L	E				30Q	U	A	L	I	T	Y	
	I											T			31P	U	L	S	E				
	32C	A	R	N	O	T	E	N	G	I	N	E				A							
												33R	E	T	A	R	D	A	T	I	O	N	
																K							

WINNERS (April 2015)

- **Vipul Ahuja**
- **Sattwik Sadhu**
- **Atriz Roy**

Solution Senders (April 2015)

- **Anubhab Banerjee**
- **Asma Saifi**

Solution Senders (March 2015)

- **Hemen Ved**
- **Poonam Sharma**

CORE CONCEPT on

Standing Waves

When two waves of identical frequency (of similar kind) travelling from opposite directions meet, the resultant wave obtained is known as standing waves.

Standing waves can either be pure standing waves or partial standing waves.

Pure Standing Waves

Here the amplitude of the two superposing waves need to be identical too.

Let the two travelling waves be

$y_1(x, t) = A\sin(\omega t - kx)$

$y_2(x, t) = A\sin(\omega t + kx)$

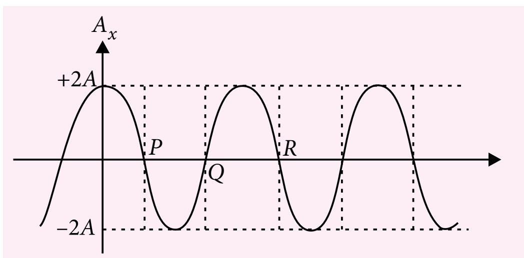
Due to their superposition, the resultant wave is

$$y_R(x, t) = y_1(x, t) + y_2(x, t)$$
$$= A[\sin(\omega t - kx) + \sin(\omega t + kx)]$$
$$= (2A\cos kx)\sin\omega t$$

where $(2A\cos kx) = A_x$ is clearly a position dependent function (cosine, hence periodic too) and is a constant for a particular location and is known as amplitude of oscillation at that location.

While the second term, $\sin(\omega t)$ indicates SHM of the particle at position x with amplitude A_x. Therefore, $\sin\omega t$ term is common for all particles, hence each particle has same frequency of oscillation. But what about the phase? Let us see.

The amplitude, $A_x = 2A\cos(kx)$ as a function of x can be plotted as below.

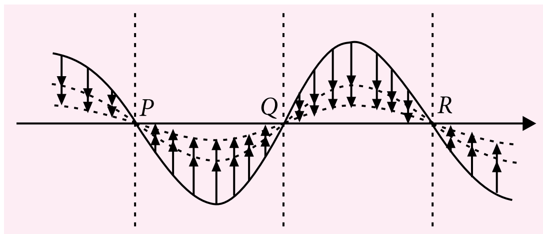


But, what would a negative amplitude indicate, as in from location P to Q on x-axis?

It first indicates that the instantaneous phase of these medium particles are 180° out of phase with respect to the particles which have a positive amplitude, as the particles located betweeen Q and R. But it should be noted that all particles located between P and Q oscillate in same phase, *i.e.* reach their extreme ends together, cross their mean position together but they have different amplitude of oscillation.

Similar is the situation with particles located between Q and R.

Let me clarify this with a more suitable pictorial representation.



Note that, when particles located between P and Q are at their lower extreme, the particles between Q and R are at their upper extreme and when one region particles are travelling up, the others are travelling down but they would cross the mean position together and twice in every oscillation. Hence 180° out of phase!

But why were they named standing waves?

The answer again lies in the amplitude expression,

$A_x = 2A\cos(kx)$

Since, its a cosine function, it will have all values between −1 to +1 including zero.

What would zero amplitude of oscillation mean?

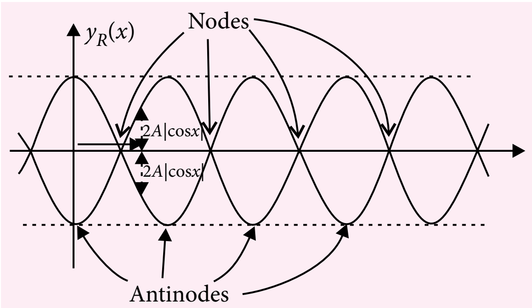
Amplitude indicates the maximum displacement from mean position and when the maximum displacement itself is zero, it means that the particles located here do not oscillate at all. Hence, we can say that these particles of zero amplitude are standing at their respective points.

These points of zero amplitude are known as nodes,

Contributed By: Bishwajit Barnwal, Aakash Institute, Kolkata

whereas the particles with maximum amplitude ($\pm 2A$) form the location of antinodes.

We always represent standing waves by drawing the envelope of oscillation of the particles of the medium, as below.



We can easily find out the location of

(i) Nodes, by using $\cos kx = 0$

$$\Rightarrow \quad kx = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}x = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \quad x = (2n+1)\frac{\lambda}{4} = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$$

i.e. at odd multiples of $\frac{\lambda}{4}$

(ii) Antinodes, by using $\cos kx = \pm 1$

$$\Rightarrow \quad kx = n\pi$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}x = n\pi$$

$$\Rightarrow \quad x = n\left(\frac{\lambda}{2}\right) = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$$

i.e. at integral multiples of $\frac{\lambda}{2}$

Hence the distance between two consecutive nodes or antinodes is $\frac{\lambda}{2}$.

Node-to-node is a loop of standing wave and hence is also said to be the loop length which is $\frac{\lambda}{2}$.

It is also important to note here that unlike travelling waves, energy is not transformed here from one end to the other. The energy is confined between node to node *i.e.* within a loop. This can easily be understood since, one wave transports energy in positive x-direction while the other transports equal amount in negative x-direction. Hence overall, there is no transfer of energy.

So, is $y_R(x, t) = (2A\cos kx)\sin(\omega t)$ the only standing wave equation?

No!

$$\cos kx \begin{cases} \sin\omega t \Rightarrow y_R(x, t) = (2A\cos kx)\sin\omega t \\ \cos\omega t \Rightarrow y_R(x, t) = (2A\cos kx)\cos\omega t \end{cases}$$

[which clearly can be obtained from superposition of two cosine travelling wave functions, $\cos(\omega t - kx)$ and $\cos(\omega t + kx)$].

$$\sin kx \begin{cases} \sin\omega t \Rightarrow y_R(x, t) = (2A\sin kx)\sin\omega t \\ \cos\omega t \Rightarrow y_R(x, t) = (2A\sin kx)\cos\omega t \end{cases}$$

Each of the above four equations form the standing waves.

$(2A\cos kx)$ or $(2A\sin kx)$ which can be used to indicate the magnitude at a location, depends on the boundary condition at $x = 0$, *i.e.* if

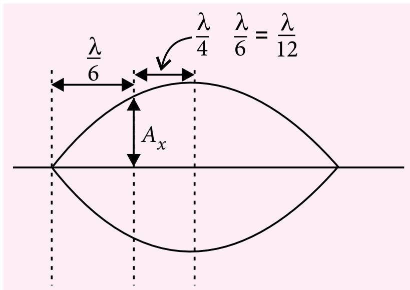
(a) at $x = 0$, node exists we use $2A\sin kx$

(b) at $x = 0$, antinode exists we use $2A\cos kx$

$\cos(\omega t)$ or $\sin(\omega t)$ will be used seeing if the particles start from mean position ($\sin\omega t$) or extreme end ($\cos\omega t$) of oscillaton at $t = 0$.

So, how to find the amplitude of oscillation at any arbitrary location, say at a distance of $\frac{\lambda}{6}$ from node?

We can use both $(2A\cos kx)$ as well as $(2A\sin kx)$ to find the amplitude, as below.



$$A_x = 2A\cos\left(k\frac{\lambda}{12}\right) \quad \text{or} \quad 2A\sin\left(k\frac{\lambda}{6}\right)$$

$$= 2A\cos\left(\frac{2\pi}{\lambda}\frac{\lambda}{12}\right) \quad \text{or} \quad 2A\sin\left(\frac{2\pi}{\lambda}\frac{\lambda}{6}\right)$$

$$= 2A\cos\left(\frac{\pi}{6}\right) \quad \text{or} \quad 2A\sin\left(\frac{\pi}{3}\right) = \sqrt{3}A$$

Since each particle executes simple harmonic motion of different amplitude, their energy of oscillation are also different, clearly maximum at antinodes.

Considering a standing wave in a stretched string as in the strings of a guitar where

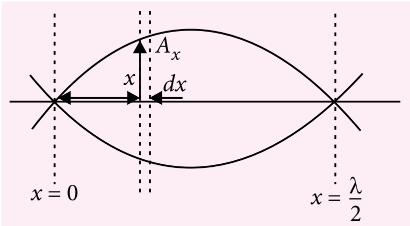
T = tension in string

μ = mass per unit length

ω = angular frequency of wave

k = wave number of wave = $\frac{2\pi}{\lambda}$

We can find the energy confined within a loop.



$$\therefore \quad dE = \frac{1}{2}(dm)\,\omega^2 A_x^2$$

[This result is a standard result of energy of a particle in simple harmonic motion]

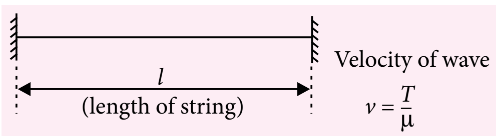
$$= \frac{1}{2}(\mu dx)\,\omega^2 (2A\sin kx)^2$$

$$\Rightarrow \quad E = 2\mu\omega^2 A^2 \int_0^{\lambda/2} \sin^2(kx)dx$$

$$= 2\mu\omega^2 A^2 \int_0^{\lambda/2} \frac{(1-\cos(2kx))}{2}dx$$

$$\therefore \quad E = \frac{\mu\omega^2 A^2 \lambda}{2}$$

Now, let us try to find out, what are the frequencies with which we can set up standing waves in a string fixed at both ends.



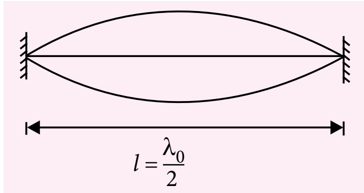
The smallest frequency with which standing waves can be set up in any system is said to be fundamental frequency (f_0). Finding f_0, clearly means having largest wavelength so that

$v = f\lambda$ = constant,

since velocity of wave is only medium dependent.

Since both the ends are fixed, they will necessarily form nodes.

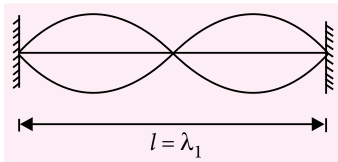
Therefore, to have largest wavelength we will have to insert one antinode between these two nodes.



$$\therefore \quad v = f_0\lambda_0 = f_0(2l)$$

$$\Rightarrow \quad f_0 = \frac{v}{2l} = \frac{\sqrt{T/\mu}}{2l}$$

Overtones are the higher frequencies, with respect to the fundamental frequency with which standing waves can be set up. For example, 1^{st} overtone is the immediate higher next frequency (hence immediate lower next wavelength). Therefore we insert one more antinode here.



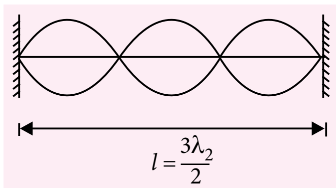
$$\therefore \quad v = f_1\lambda_1$$

$$\Rightarrow \quad f_1 = \left(\frac{v}{2l}\right)2 = 2f_0$$

$\therefore$ 1^{st} overtone = $2f_0$ = 2^{nd} harmonic

(Note : All integral multiples of fundamental frequency are said to be harmonics of f_0).

Similarly for finding 2^{nd} overtone,



$$\therefore \quad v = f_2\lambda_2$$

$$\Rightarrow \quad f_2 = \frac{3v}{2l} = 3f_0$$

$\Rightarrow$ 2^{nd} overtone = 3^{rd} harmonic

$\therefore$ Generalising, we have, n^{th} overtone

$f_n = (n + 1)^{th}$ harmonic

$$= (n+1)f_0 = (n+1)\frac{v}{2l}$$

If the string oscillates in m loops, the frequency of oscillation is

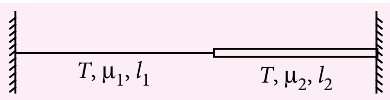
$$f_m = mf_0 = m\frac{v}{2l} = m\frac{\sqrt{T/\mu}}{2l}$$

We can easily change the frequency of oscillation by changing T (by increasing/decreasing tension), μ (thin/thick wire) or l (long/short wire). This is the basic of sound production of different frequency in guitar wires. By the movement of the fingers, on the string one controls the location of nodes whereas by plucking it with the other hand, we control the location of antinodes.

If the string's tension is increased/decreased slightly (less than 5%), we can apply error formula to find the percentage change in frequency of string

i.e. $\frac{\Delta f}{f} \approx \frac{1}{2}\frac{\Delta T}{T}$

Standing Waves in Composite Strings



A light string is attached to a heavy string, with a common tension T.

If $\mu_2 > \mu_1$, then the velocity of wave $\left(v = \sqrt{\frac{T}{\mu}}\right)$,

$v_1 > v_2$

and as $v = f\lambda$, the strings are supposed to oscillate with same frequency, therefore,

$\lambda_1 > \lambda_2$

i.e. thin string will have larger wavelength and hence lesser loops.

If p and q be the number of loops in the thin and thick strings respectively, the junction obviously being a node.

$$f_1 = f_2$$

$$\Rightarrow \quad p\frac{v_1}{2l_1} = q\frac{v_2}{2l_2}$$

$$\Rightarrow \quad p\frac{\sqrt{T/\mu_1}}{l_1} = q\frac{\sqrt{T/\mu_2}}{l_2}$$

Partial Standing Waves

This is a very less talked about topic. If the two superposing waves travelling from opposite directions are of different amplitudes, we obtain a partial standing wave.

The discussion below will clarify the name.

Let, $y_1(x, t) = A_1\sin(\omega t - kx)$

$y_2(x, t) = A_2\sin(\omega t + kx) \qquad (A_2 < A_1)$

$\therefore$ Resultant wave,

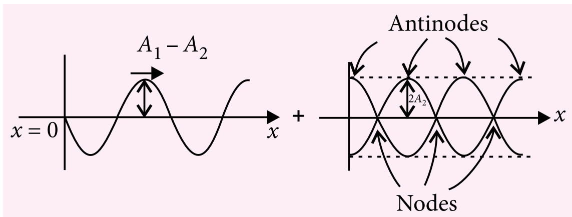
$y_R(x, t) = A_1\sin(\omega t - kx) + A_2\sin(\omega t + kx)$

$\Rightarrow \quad y_R(x, t) = (A_1 - A_2)\sin(\omega t - kx) + A_2\sin(\omega t - kx) + A_2\sin(\omega t + kx)$

In this expression, clearly

$A_2\sin(\omega t - kx) + A_2\sin(\omega t + kx)$ will form standing wave.

$$\therefore \quad y_R(x,t) = \underbrace{(A_1 - A_2)\sin(\omega t - kx)}_{\text{Travelling wave transporting energy in +ve } x\text{-direction}} + \underbrace{(2A_2\cos kx)\sin\omega t}_{\text{Pure standing wave}}$$

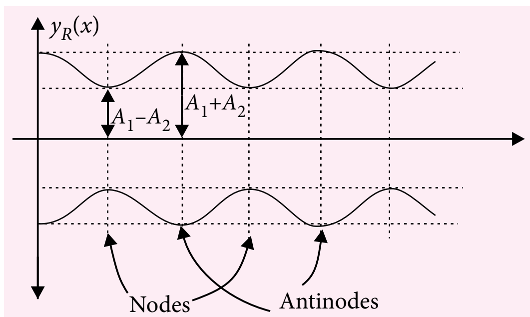


Clearly, a travelling wave of amplitude $A_1 - A_2$ is superposed over a pure standing wave. Hence the amplitude of each point is raised by $A_1 - A_2$.

$\therefore$ At nodes, $A = A_1 - A_2 + 0 = A_1 - A_2 = A_{min}$

At antinodes, $A = (A_1 - A_2) + 2A_2 = A_1 + A_2 = A_{max}$

The resultant envelope of the wave in this case would look like



"An experiment is a question which science poses to Nature, and a measurement is the recording of Nature's answer."

-Max Planck

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

Q1. When you load up a plastic shopping bag with groceries and then carry the bag by the loops at the top of the bag, why will the loops initially withstand the load but then, several minutes later, begin to stretch, perhaps to the point of tearing?

– Saumya Shah (W.B.)

Ans. If you suspend a load from the lower end of a spring hanging from a ceiling, the spring will stretch by a certain amount and then stay stretched. Plastic, which consists of polymers, is different. If you suspend a load from the lower end of a plastic strip, the strip will initially stretch like the spring but thereafter it will gradually stretch more which is called viscoelastic creep. The mechanism of this creep can vary from polymer to polymer. The polymer consists of many long and entangled molecules. When the polymer is put under load, these molecules gradually disentangle somewhat because they are pulled in the direction of the load. The gradual reorientation of the molecules allows the plastic to gradually stretch. If the plastic stretches enough, it may also get narrowed perpendicular to the direction of the load called as necking.

Q2. What keeps a kite aloft and what determines a stable flight, as opposed to a chaotic one in which the kite constantly loops and flutters?

– Aman Gupta (U.P.)

Ans. A triangular kite is a flexible surface that faces into the wind while tilted up at an angle, said to be the angle of attack. Four forces act on the kite are (i) The gravitational force pulls downward. (ii) Because the wind is deflected downward by the kite's face, the kite experiences an upward lift. (iii) The wind also produces a drag force in the direction of the wind. (iv) The string produces a force that is downward and into the wind.

If the kite is not in stable flight, the torques due to the forces automatically rotate the kite around the bridle point (the point where the main, long string branches into separate strings running to various points on the kite frame). The rotation changes the kite's angle of attack and thus alters the lift and drag. As a result, the kite not only rotates but also moves vertically. The vertical motion changes the angle at which the string pulls on the bridle point and thus also the horizontal and vertical pulls of the string.

Stable flight occurs if three quantities vanish (i) the torques, (ii) the net vertical force, and (iii) the net horizontal force. For these to vanish, not only must the kite have the proper orientation but also the string must pull at the proper angle and with the proper force. The kite is then said to be in an equilibrium state. For a given wind speed, there might be more than one equilibrium state. If the wind changes, both the orientation of the kite and the angle of the string must change for the kite to find a new equilibrium state.

Q3. How is an aircraft protected from lightning while in the air?

– Abhishek Thomas (Kerala)

Ans. Thunderclouds can be identified, and the first precaution is to ensure that the aircraft stays away from these highly charged clouds. Aircraft are fitted with conducting brushes at the tips of wings and the rudder to ensure that they stay close to the electrical potential of the air mass in which they are flying. The electrical potential in the atmosphere varies significantly as a function of altitude. In addition, just the movement of the airplane through the air mass can be fast enough to produce electrostatic charge. This charge must be continuously dissipated. In comparison with a large cloud or even the ground, an aircraft does not appear an attractive target for a lightning bolt. It is, after all, fairly well insulated from the ground. Even if there is flow of high current through the skin of the aircraft,the highly conducting metallic construction ensures that passengers are protected from the effect of electrical activity outside. Communication system could be affected. The chance of an accident appears greater than the actual observed incidence. This must be a reflection of the precautions taken by pilots. In spite of the brushes at the tips of wings, an aircraft can accumulate significant electrical charge on its body prior to landing. Sparks or discharges must be prevented. In the earlier days, airplanes used to deploy a metal chain that trailed and touched the ground before the wheels of the plane did. Modern aircraft use tyres that are made of conducting rubber.

TARGET PMTs

Useful for all National and State Level PMTs

Reminder!

- AIPMT : 3rd May
- WB-JEE : 5th & 6th May
- MH-CET : 7th May
- COMEDK : 10th May
- AIIMS : 1st June

1. The major contribution of Sir C.V. Raman is
(a) explanation of photoelectric effect
(b) principle of buoyancy
(c) scattering of light by molecules of a medium
(d) electromagnetic theory

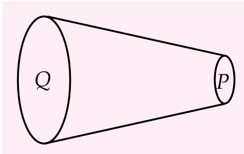
2. The period of revolution of Jupiter around the Sun is 12 times the period of revolution of the Earth around the Sun. The distance between the Jupiter and Sun is n times the distance between the Earth and Sun. Then the value of n is
(a) $(144)^{3/2}$ (b) $(144)^{2/3}$
(c) $\sqrt[3]{144}$ (d) $\sqrt[4]{144}$

3. A source of sound producing wavelength of 50 cm is moving away from a stationary observer with $(1/5)^{\text{th}}$ speed of sound. Then, what is the wavelength of sound heard by the observer?
(a) 70 cm (b) 55 cm
(c) 40 cm (d) 60 cm

4. A long straight wire along the z-axis carries a current I in the negative z-direction. The magnetic field $\vec{B}$ at a point having coordinates (x, y) in the $z = 0$ plane is
(a) $\dfrac{\mu_0 I(x\hat{i} - y\hat{j})}{2\pi(x^2+y^2)}$ (b) $\dfrac{\mu_0 I(x\hat{j} - y\hat{i})}{2\pi(x^2+y^2)}$
(c) $\dfrac{\mu_0 I(x\hat{i} + y\hat{j})}{2\pi(x^2+y^2)}$ (d) $\dfrac{\mu_0 I(y\hat{i} - x\hat{j})}{2\pi(x^2+y^2)}$

5. If the absolute errors in two physical quantities A and B are a and b respectively, then the absolute error in the value of $A - B$ is
(a) $a - b$ (b) $b - a$
(c) $a \pm b$ (d) $a + b$

6. A conductor has a non-uniform section as shown in the figure. A steady current is flowing through it. Then the drift speed of the electrons



(a) varies unpredictably
(b) increases from P to Q
(c) decreases from P to Q
(d) is constant throughout the conductor

7. A 0.01 H inductor and $\sqrt{3}\pi\,\Omega$ resistance are connected in series with a 220 V, 50 Hz ac source. The phase difference between the current and the voltage is
(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{6}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{4}$

8. A man moves 20 m north, then 10 m east and then $10\sqrt{2}$ m south-west. His displacement is
(a) 20 m north (b) $10\sqrt{2}$ m north-west
(c) 10 m north (d) $10\sqrt{2}$ m south-east

9. When a light of photons of energy 4.2 eV is incident on a metallic sphere of radius 10 cm and work function 2.4 eV, photoelectrons are emitted. The number of photoelectrons liberated before the emission is stopped, is
$\left(e = 1.6 \times 10^{-19}\text{ C and } \dfrac{1}{4\pi\varepsilon_0} = 9\times10^9\text{ N m}^2\text{ C}^{-2}\right)$
(a) 6.25×10^8 (b) 1.25×10^{18}
(c) 1.25×10^8 (d) 6.25×10^{18}

10. At time $t = 0$, two bodies A and B are at the same point. A moves with constant velocity v and B starts from rest and moves with constant acceleration. The relative velocity of B with respect to A when the bodies meet each other is
(a) $2v$ (b) $\dfrac{v}{2}$ (c) $\dfrac{v}{3}$ (d) v

11. For most materials the Young's modulus is n times the rigidity modulus, where n is
(a) 2 (b) 3 (c) 4 (d) 5

12. Choose the incorrect statement.
(a) No work is done if the displacement is perpendicular to the direction of the applied force.
(b) If the angle between the force and displacement vectors is obtuse, then the work done is negative.
(c) Frictional force is non-conservative.
(d) All the central forces are non-conservative.

13. A pipe closed at one end and open at the other end resonates with a sound of frequency 135 Hz and also with 165 Hz, but not at any other frequency intermediate between these two. Then, the frequency of the fundamental note of the pipe is
(a) 15 Hz (b) 60 Hz
(c) 7.5 Hz (d) 30 Hz

14. Choose the correct statement.
(a) Polar molecules have permanent electric dipole moment.
(b) CO_2 molecule is a polar molecule.
(c) H_2O is a non-polar molecule.
(d) The dipole field at large distances falls off as $\frac{1}{r^2}$.

15. The linear momentum of a particle varies with time t as $p = a + bt + ct^2$ where a, b and c are constants. Then which of the following is correct?
(a) Force is dependent linearly on time.
(b) Velocity of particle is inversely proportional to time.
(c) Displacement of the particle is independent of time.
(d) Force varies with time in a quadratic manner.

16. A mass M is suspended from a light spring. An additional mass m is added, displaces the spring further by a distance x. Now the combined mass will oscillate with a period
(a) $T = 2\pi\sqrt{\frac{mg}{x(M+m)}}$
(b) $T = 2\pi\sqrt{\frac{(M+m)x}{mg}}$
(c) $T = 2\pi\sqrt{\frac{mgx}{(M+m)}}$
(d) $T = 2\pi\sqrt{\frac{(M+m)}{mgx}}$

17. A stone tied to a rope is rotated in a vertical circle with uniform speed. If the difference between the maximum and minimum tensions in the rope is 20 N, mass of the stone is (g = 10 m s^{-2})
(a) 2 kg (b) 1 kg
(c) 1.5 kg (d) 0.5 kg

18. A charged particle with velocity $\vec{v} = x\hat{i} + y\hat{j}$ moves in a magnetic field $\vec{B} = y\hat{i} + x\hat{j}$. The magnitude of the force acting on the particle is F. The correct option for F is
(i) No force will act on the particle if $x = y$.
(ii) Force will act along y axis if $y < x$.
(iii) Force is proportional to $(x^2 - y^2)$ if $x > y$.
(iv) Force is proportional to $(x^2 + y^2)$ if $y > x$.
(a) (i) and (ii) are true (b) (i) and (iii) are true
(c) (ii) and (iv) are true (d) (iii) and (iv) are true

19. An astronomical telescope arranged for normal adjustment has a magnification of 6. If the length of the telescope is 35 cm, then the focal lengths of objective and eyepiece respectively are
(a) 30 cm, 6 cm (b) 30 cm, 5 cm
(c) 5 cm, 30 cm (d) 40 cm, 5 cm

20. Two deuterium nuclei each of mass m, fuse together to form a helium nucleus, releasing an energy E. If c is the velocity of light, the mass of helium nucleus formed will be
(a) $2m + \frac{E}{c^2}$ (b) $\frac{E}{mc^2}$
(c) $m + \frac{E}{c^2}$ (d) $2m - \frac{E}{c^2}$

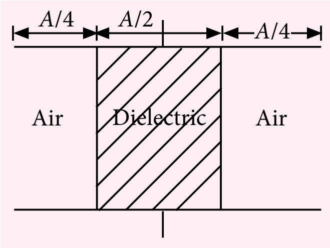
21. How much heat energy must be supplied to 14 g of nitrogen at room temperature to raise its temperature by 40°C at constant pressure?
(Molecular mass of N_2 = 28 g,
R = universal gas constant)
(a) 50R (b) 60R (c) 70R (d) 80R

22. An electrical device which offers a low resistance to the current in one direction but a high resistance to the current in opposite direction is
(a) current amplifier (b) oscillator
(c) power amplifier (d) rectifier

23. A copper wire and a steel wire of the same length and same cross-section are joined end to end to form a composite wire. The composite wire is hung from a rigid support and a load is suspended from the other end. If the increase in length of the composite wire is 2.4 mm, then the increase in lengths of steel and copper wires are
($Y_{Cu} = 10 \times 10^{10}$ N m^{-2}, $Y_{steel} = 2 \times 10^{11}$ N m^{-2})

(a) 0.4 mm, 2.0 mm (b) 1.2 mm, 1.2 mm
(c) 0.6 mm, 1.8 mm (d) 0.8 mm, 1.6 mm

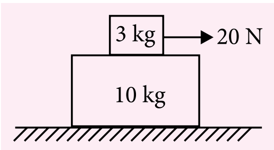
24. Consider a parallel plate capacitor of capacitance 10 μF filled with air. When the gap between the plates is filled partly with a dielectric of dielectric constant 4,



as shown in figure, the new capacitance of the capacitor is (A is the area of each plate)
(a) 20 μF (b) 40 μF
(c) 2.5 μF (d) 25 μF

25. The objective lens of an optical instrument is an achromatic combination with a focal length of 90 cm. The two lenses possess dispersive powers 0.024 and 0.036 respectively and are in contact with each other. Then their focal lengths are
(a) –30 cm, 45 cm (b) 45 cm, 30 cm
(c) 30 cm, –45 cm (d) 30 cm, –30 cm

26. A 3 kg block is placed over a 10 kg block and both are placed on a smooth horizontal surface. The coefficient



of friction between the blocks is 0.2. If a horizontal force of 20 N is applied to 3 kg block, accelerations of the two blocks in m s^{-2} are (g = 10 m s^{-2})
(a) $\frac{13}{4}, 0.6$ (b) $\frac{14}{3}, 3$
(c) $\frac{13}{4}, 3$ (d) $\frac{14}{3}, 0.6$

27. A flash light lamp is marked 3.5 V and 0.28 A. The filament temperature is 425°C. The filament resistance at 0°C is 4 Ω. Then, the temperature coefficient of resistance of the material of the filament is
(a) 8.5×10^{-3} °C^{-1} (b) 3.5×10^{-3} °C^{-1}
(c) 0.5×10^{-3} °C^{-1} (d) 5×10^{-3} °C^{-1}

28. If in an amplitude modulated wave, the maximum amplitude is 10 V and the modulation index is 2/3, then the minimum amplitude is
(a) 2 V (b) 7 V (c) 9 V (d) 6 V

29. A body of mass 300 kg is moved through 10 m along a smooth inclined plane of angle 30°. The work done in moving is (g = 9.8 m s^{-2})
(a) 4900 J (b) 9800 J
(c) 14700 J (d) 2450 J

30. A plane electromagnetic wave travels in free space. Then the ratio of the magnitudes of electric and magnetic fields at a point is equal to
(a) energy of electromagnetic wave.
(b) inverse of the velocity of the electromagnetic wave.
(c) inverse of the energy of electromagnetic wave.
(d) velocity of electromagnetic wave.

31. The pressure P for a gas is plotted against its absolute temperature T for two different volumes V_1 and V_2 where $V_1 > V_2$. If P is plotted on y-axis and T on x-axis, then
(a) the curve for V_1 has greater slope than that for V_2.
(b) the curve for V_2 has greater slope than that for V_1.
(c) both curves have same slope.
(d) the curves intersect at some point other than $T = 0$.

32. A paramagnetic sample shows a net magnetization of 0.8 A m^{-1}, when placed in an external magnetic field of strength 0.8 T at a temperature 5 K. When the same sample is placed in an external magnetic field of 0.4 T at a temperature of 20 K, the magnetization is
(a) 0.1 A m^{-1} (b) 0.2 A m^{-1}
(c) 0.4 A m^{-1} (d) 0.8 A m^{-1}

33. A mass is suspended from the end of a spring. When the system is oscillating the amplitude of oscillation is 4 cm and the maximum kinetic energy of oscillation of the system is 1 J. Then the force constant of the spring is
(a) 2500 N m^{-1} (b) 1250 N m^{-1}
(c) 500 N m^{-1} (d) 250 N m^{-1}

34. A p-n-p transistor is used in common emitter mode in an amplifier circuit. When base current is changed by an amount ΔI_B, the collector current changes by 2 mA. If the current amplification factor is 50, then the value of ΔI_B is
(a) 15 μA (b) 40 μA
(c) 50 μA (d) 60 μA

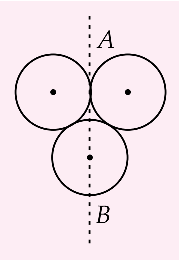
35. A shell is fired from a cannon with a velocity v at an angle θ with the horizontal. At the highest point in its path it explodes into two pieces of equal masses. One of the pieces retraces its path and reaches the cannon. Then the velocity of the other piece immediately after the explosion is
(a) $2v\cos\theta$ (b) $\frac{3}{2}v\cos\theta$
(c) $3v\cos\theta$ (d) $2v\sin\theta$

36. A metal rod of length 2 m is rotating with an angular velocity of 100 rad s^{-1} in a plane perpendicular to a uniform magnetic field of 0.3 T. The potential difference between the ends of the rod is
(a) 30 V (b) 40 V (c) 60 V (d) 100 V

37. Displacement of a body is $(5\hat{i}+3\hat{j}-4\hat{k})$ m when a force $(6\hat{i}+6\hat{j}+4\hat{k})$ N acts for 5 s. The power is
(a) 1.6 W (b) 9.6 W (c) 6.4 W (d) 3.2 W

38. If the wavelength of light that is emitted from hydrogen atom when an electron falls from orbit n = 2 to orbit n = 1 is 122 nm, then minimum wavelength of the series is
(a) 405 Å (b) 9150 Å
(c) 812 Å (d) 915 Å

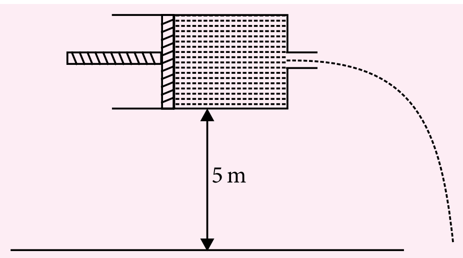
39. Three uniform circular discs, each of mass M and radius R are kept in contact with each other as shown in the figure. Moment of inertia of the system about the axis AB is



(a) $\frac{7}{4}MR^2$ (b) $\frac{11}{4}MR^2$
(c) $\frac{11}{2}MR^2$ (d) $\frac{MR^2}{4}$

40. A glass slab of thickness 8 cm contains the same number of waves as 10 cm long path of water when both are traversed by the same monochromatic light. If the refractive index of water is $\frac{4}{3}$, the refractive index of glass is
(a) $\frac{5}{3}$ (b) $\frac{5}{4}$ (c) $\frac{16}{15}$ (d) $\frac{3}{2}$

41. A syringe of diameter 1 cm having a nozzle of diameter 1 mm, is placed horizontally at a height 5 m from the ground as shown below. An incompressible non-viscous liquid is filled in the syringe and the liquid is compressed by moving the piston at a speed of 0.5 m s^{-1}, the horizontal distance travelled by the liquid jet is
(g = 10 m s^{-2})



(a) 12.5 m (b) 25 m (c) 50 m (d) 75 m

42. A radioactive substance has density ρ, volume V and decay constant λ. If the molecular weight of the substance is M, and Avogadro number is N_A, then the activity of the substance after time t is
(a) $\left(\frac{\lambda V\rho N_A}{M}\right)(1-e^{-\lambda t})$
(b) $\left(\frac{N_A V}{\rho M}\right)e^{-\lambda t/2}$
(c) $\left(\frac{\lambda N_A}{V\rho M}\right)e^{-\lambda t}$ (d) $\left(\frac{\lambda V\rho N_A}{M}\right)e^{-\lambda t}$

43. Two polarizers have their axes inclined at 45° to each other. If unpolarized light of intensity I_0 is incident on the first polarizer, then the intensity of light transmitted through second polarizer is
(a) $\frac{I_0}{4}$ (b) $\frac{I_0}{2}$ (c) I_0 (d) 0

44. An infinitely long thin straight wire has uniform linear charge density of $\frac{1}{3}$C m^{-1}. Then, the magnitude of the electric intensity at a point 18 cm away is
(a) 0.33×10^{11} N C^{-1} (b) 3×10^{11} N C^{-1}
(c) 0.66×10^{11} N C^{-1} (d) 1.32×10^{11} N C^{-1}

45. The thermal conductivity of a rod depends on
(a) length (b) area of cross-section
(c) mass (d) material of the rod

SOLUTIONS

1. **(c) :** The major contribution of Sir C.V. Raman is scattering of light by molecules of a medium.

2. **(c) :** According to Kepler's third law,
The period of revolution of a planet around the Sun is

$$T=\frac{2\pi r^{3/2}}{\sqrt{GM_S}} \text{ or } T^2=\frac{4\pi^2 r^3}{GM_S}.$$

where M_S is the mass of the Sun and r is the radius of orbit.

$\therefore \frac{T_J^2}{T_E^2}=\frac{r_{JS}^3}{r_{ES}^3}$, where r_{JS} is the distance between the Jupiter and Sun and r_{ES} is the distance between the Earth and Sun.

Here, $T_J = 12T_E$, $r_{JS} = nr_{ES}$

$$\therefore \left(\frac{12T_E}{T_E}\right)^2=\left(\frac{nr_{ES}}{r_{ES}}\right)^3$$

$$144 = n^3 \text{ or } n=\sqrt[3]{144}$$

3. **(d) :** Here,
Wavelength of the source, λ = 50 cm

Speed of sound = v

Speed of the source, $v_s = \frac{1}{5}v$

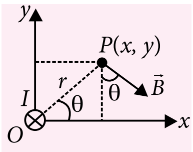
As the source is moving away from the stationary observer, so wavelength of sound heard by the observer is

$$\lambda' = \left(\frac{v+v_s}{v}\right)\lambda = \left(\frac{v+\frac{1}{5}v}{v}\right)\lambda$$

$$= \frac{6}{5}\lambda = \frac{6}{5}(50\text{ cm}) = 60\text{ cm}$$

4. **(d) :** The wire carries the current I in the negative z-direction.

The magnetic field $\vec{B}$ at point P is perpendicular to OP in the direction shown in figure.



$\therefore \quad \vec{B} = B_0 \sin\theta\hat{i} - B_0\cos\theta\hat{j}$

Here, $B_0 = \frac{\mu_0 I}{2\pi r}$

$\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

$\therefore \quad \vec{B} = \frac{\mu_0 I}{2\pi r^2}(y\hat{i} - x\hat{j})$

$= \frac{\mu_0 I(y\hat{i} - x\hat{j})}{2\pi(x^2+y^2)}$ (as $r^2 = x^2 + y^2$)

5. **(d) :** The absolute error in the difference of two physical quantities is equal to the sum of the absolute errors in the individual quantities.

$\therefore$ The absolute error in $A - B$ is $= a + b$

6. **(c) :** The drift speed is $v_d = \frac{I}{neA}$

where n is the number density of electrons and A is the area of cross-section of the conductor.

For steady current, $v_d \propto \frac{1}{A}$

As A increases from P to Q, so v_d decreases from P to Q.

7. **(b) :** Here,

$L = 0.01$ H, $R = \sqrt{3}\pi\,\Omega$, $\upsilon = 50$ Hz

The inductive reactance is

$X_L = \omega L = 2\pi\upsilon L$

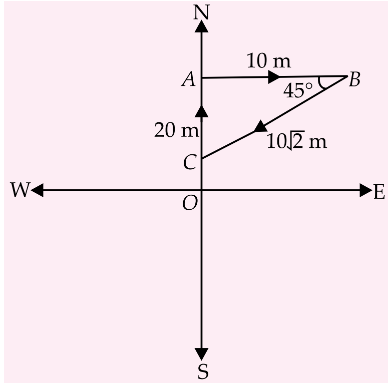
$= 2\pi(50\text{ Hz})(0.01\text{ H}) = \pi\ \Omega$

The phase difference ϕ between the current and the voltage is

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$ $\left(\text{as } \tan\phi = \frac{X_L}{R}\right)$

$= \tan^{-1}\left(\frac{\pi\ \Omega}{\sqrt{3}\pi\ \Omega}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30° = \frac{\pi}{6}$

8. **(c) :** The situation is shown in figure.



$\overrightarrow{OA}$ represents 20 m north, $\overrightarrow{AB}$ represents 10 m east and $\overrightarrow{BC}$ represents $10\sqrt{2}$ m south-west.

If $\hat{i}$ and $\hat{j}$ be the unit vectors along east and north respectively, then

$\overrightarrow{OA} = 20\hat{j}$ m, $\overrightarrow{AB} = 10\hat{i}$ m

$\overrightarrow{BC} = (-10\sqrt{2}\cos 45°\hat{i} - 10\sqrt{2}\sin 45°\hat{j})$ m

$= \left(-10\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\hat{i} - 10\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\hat{j}\right)$ m

$= (-10\hat{i} - 10\hat{j})$ m

$\therefore$ Displacement $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{OC} = 20\hat{j}$ m $+ 10\hat{i}$ m $+ (-10\hat{i} - 10\hat{j})$ m

$= 10\hat{j}$ m $= 10$ m due north

9. **(c) :** Here,

Radius of the sphere, $r = 10$ cm $= 10 \times 10^{-2}$ m

Work function, $\phi_0 = 2.4$ eV

Energy of a photon, $E = h\upsilon = 4.2$ eV

According to Einstein's photoelectric equation

$K_{max} = h\upsilon - \phi_0$

But $K_{max} = eV_0$

where V_0 is the stopping potential.

$\therefore \quad eV_0 = h\upsilon - \phi_0$

$eV_0 = 4.2$ eV $- 2.4$ eV $= 1.8$ eV

or $\quad V_0 = 1.8$ V

The sphere will stop emitting photoelectrons, when the potential on its surface becomes 1.8 V.

Let n be the number of photoelectrons emitted from the sphere. Then charge on the sphere is $q = ne$

Potential on the surface of the sphere is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{ne}{r}$$

$$\therefore \quad n = \frac{Vr}{\frac{1}{4\pi\varepsilon_0}e} = \frac{(1.8\text{ V})(10\times10^{-2}\text{ m})}{(9\times10^9\text{ N m}^2\text{ C}^{-2})(1.6\times10^{-19}\text{ C})}$$

$= 1.25 \times 10^8$

10. (d) : Let the two bodies A and B meet at time t.
Distance travelled by A in time t,

$s_A = vt$

Distance travelled by B in time t,

$$s_B = \frac{1}{2}at^2 \quad (\because u = 0)$$

At the time of meeting,

$s_A = s_B$

$$\therefore \quad vt = \frac{1}{2}at^2 \text{ or } a = \frac{2v}{t} \quad \text{...(i)}$$

Velocity of A after time t, $v_A = v$
Velocity of B after time t,

$$v_B = at = \left(\frac{2v}{t}\right)t = 2v \quad \text{(using (i))}$$

$\therefore$ The relative velocity of B with respect to A when the bodies meet each other is

$v_{BA} = v_B - v_A = 2v - v = v$

11. (b) : For most materials, the modulus of rigidity η is one third of the Young's modulus Y.

i.e. $\eta = \frac{1}{3}Y$ or $Y = 3\eta$ $\therefore$ $n = 3$

12. (d) : All the central forces are conservative.

13. (d)

14. (a) : CO_2 is a non-polar molecule.
H_2O is a polar molecule.
The dipole field at large distances falls off as $\frac{1}{r^3}$.

15. (a) : Here,
Linear momentum, $p = a + bt + ct^2$

As force, $F = \frac{dp}{dt}$

$$\therefore \quad F = \frac{d}{dt}(a + bt + ct^2)$$

$F = b + 2ct$

16. (b) : Let k be spring constant of the spring.
When the mass M is suspended from the spring, let it be stretched by distance y.

$\therefore \quad Mg = ky$ ...(i)

Now when the additional mass m is added to it, it stretches further by the distance x.

$\therefore \quad (M + m)g = k(x + y)$ ...(ii)

Subtracting eqn. (i) from eqn. (ii), we get

$(M + m)g - Mg = k(x + y) - ky$

$mg = kx$ or $k = \frac{mg}{x}$ ...(iii)

The period of the combined mass is

$$T = 2\pi\sqrt{\frac{(M+m)}{k}} = 2\pi\sqrt{\frac{(M+m)}{(mg/x)}} \quad \text{(using (iii))}$$

$$= 2\pi\sqrt{\frac{(M+m)x}{mg}}$$

17. (b) : Let m be the mass of the stone.
As $T_{max} - T_{min} = 2mg$

$$\therefore \quad m = \frac{T_{max} - T_{min}}{2g} = \frac{20\text{ N}}{2(10\text{ m s}^{-2})} = 1\text{ kg}$$

18. (b) : The force acting on a charged particle of charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is

$\vec{F} = q(\vec{v} \times \vec{B})$

Here, $\vec{v} = x\hat{i} + y\hat{j}$ and $\vec{B} = y\hat{i} + x\hat{j}$

$\therefore \quad \vec{F} = q[(x\hat{i} + y\hat{j}) \times (y\hat{i} + x\hat{j})]$

$= q(x^2 - y^2)\hat{k}$

The magnitude of the force is

$$F = \sqrt{[q(x^2 - y^2)]^2} = q(x^2 - y^2)$$

(i) If $x = y$, then $F = 0$
(iii) If $x > y$, then $F \propto (x^2 - y^2)$

19. (b) : Let f_o and f_e be focal lengths of objective and eyepiece respectively.
For normal adjustment,

Magnification of the telescope, $m = \frac{f_o}{f_e}$

and length of the telescope, $L = f_o + f_e$
As $m = 6$ and $L = 35$ cm (given)

$$\therefore \quad 6 = \frac{f_o}{f_e} \text{ or } f_o = 6f_e \quad \text{... (i)}$$

and 35 cm $= f_o + f_e$ ... (ii)

On solving eqns. (i) and (ii), we get

$f_o = 30$ cm and $f_e = 5$ cm

20. (d) : The given reaction is

$${}_1^2H + {}_1^2H \rightarrow {}_2^4He$$

The energy released in the reaction is

$E = (m + m - m_{He})c^2$

$E = (2m - m_{He})c^2$

$$\frac{E}{c^2} = 2m - m_{He} \text{ or } m_{He} = 2m - \frac{E}{c^2}$$

21. (c) : Here,
Mass of N_2, $m = 14$ g
Molecular mass of N_2, $M = 28$ g

Number of moles of N_2, $n = \frac{m}{M} = \frac{14\text{ g}}{28\text{ g}} = \frac{1}{2}$

Rise in temperature, $\Delta T = 40°C$
The amount of heat supplied at constant pressure is

$\Delta Q = nC_P\Delta T$

As nitrogen is diatomic, its molar specific heat at constant pressure is

$$C_P = \frac{7}{2}R$$

$\therefore\ \Delta Q = \frac{1}{2}\left(\frac{7}{2}R\right)(40°C) = 70R$

22. (d)

23. (d) : As same load (say W) is being applied on both the wires, which have same area of cross-section A, so stress is same for both wires.
As stress = Young's modulus × strain

$$\therefore\ \frac{W}{A} = Y_{Cu}\frac{\Delta L_{Cu}}{L_{Cu}} = Y_{steel}\frac{\Delta L_{steel}}{L_{steel}}$$

$$\frac{\Delta L_{Cu}}{\Delta L_{steel}} = \frac{L_{Cu}}{L_{steel}}\frac{Y_{steel}}{Y_{Cu}}$$

Here, $L_{Cu} = L_{steel}$, $Y_{Cu} = 10 \times 10^{10}\ N\ m^{-2}$

$Y_{steel} = 2 \times 10^{11}\ N\ m^{-2}$

$$\therefore\ \frac{\Delta L_{Cu}}{\Delta L_{steel}} = \frac{Y_{steel}}{Y_{Cu}} = \frac{2\times 10^{11}\ N\ m^{-2}}{10\times 10^{10}\ N\ m^{-2}} = 2$$

$$\Delta L_{Cu} = 2\Delta L_{steel} \quad \text{... (i)}$$

The increase in length of the composite wire is

$= \Delta L_{Cu} + \Delta L_{steel}$

But $\Delta L_{Cu} + \Delta L_{steel} = 2.4$ mm (given) ... (ii)

$\therefore\ 2\Delta L_{steel} + \Delta L_{steel} = 2.4$ mm (using (i))

$3\Delta L_{steel} = 2.4$ mm or $\Delta L_{steel} = 0.8$ mm

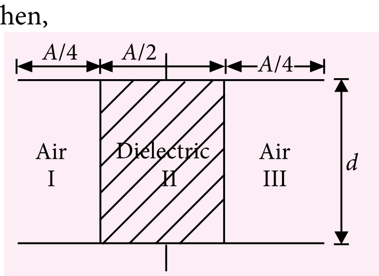
Putting this value in eqn. (i), we get

$\Delta L_{Cu} = 2(0.8\text{ mm}) = 1.6$ mm

24. (d) : Capacitance of a parallel plate capacitor filled with air is

$$C = \frac{\varepsilon_0 A}{d} = 10\ \mu F\ \text{(given)} \quad \text{... (i)}$$

where d is the distance between the plates.
When the gap between the plates is filled partly with a dielectric of dielectric constant $K(= 4)$ as shown in figure. Then,



Capacitance of part I is

$$C_1 = \frac{\varepsilon_0 (A/4)}{d} = \frac{\varepsilon_0 A}{4d}$$

Capacitance of part II is

$$C_2 = \frac{K\varepsilon_0 (A/2)}{d} = \frac{K\varepsilon_0 A}{2d}$$

Capacitance of part III is

$$C_3 = \frac{\varepsilon_0 (A/4)}{d} = \frac{\varepsilon_0 A}{4d}$$

$\because\ C_1$, C_2 and C_3 are in parallel.
$\therefore$ The new capacitance of the capacitor is
$C_{new} = C_1 + C_2 + C_3$

$$= \frac{\varepsilon_0 A}{4d} + \frac{K\varepsilon_0 A}{2d} + \frac{\varepsilon_0 A}{4d} = \frac{\varepsilon_0 A}{2d} + \frac{K\varepsilon_0 A}{2d}$$

$$= \frac{\varepsilon_0 A}{2d}(1+K) = \frac{10\ \mu F}{2}(1+4) \quad \text{(using (i))}$$

$= 25\ \mu F$

25. (c) : Let f_1 and f_2 be focal lengths of the two lenses respectively.
If F be focal length of the combination, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

As $F = 90$ cm (given)

$$\therefore\ \frac{1}{90\text{ cm}} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{... (i)}$$

For an achromatic combination,

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

where ω_1 and ω_2 be their dispersive powers.
As $\omega_1 = 0.024$ and $\omega_2 = 0.036$ (given)

$$\therefore\ \frac{0.024}{f_1} + \frac{0.036}{f_2} = 0$$

$$\frac{0.024}{f_1} = -\frac{0.036}{f_2}$$

$$f_2 = -\frac{3}{2}f_1 \quad \text{... (ii)}$$

Putting this value of f_2 in eqn. (i), we get

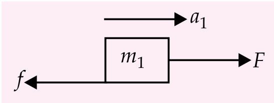
$$\frac{1}{90\text{ cm}} = \frac{1}{f_1} - \frac{2}{3f_1} = \frac{3-2}{3f_1} = \frac{1}{3f_1}$$

$$f_1 = \frac{90}{3}\text{ cm} = 30\text{ cm}$$

Putting this value of f_1 in eqn. (ii), we get

$$f_2 = -\frac{3}{2}(30\text{ cm}) = -45\text{ cm}$$

26. (d) : Here,
Mass of upper block, $m_1 = 3$ kg
Mass of lower block, $m_2 = 10$ kg
Horizontal force, $F = 20$ N
Coefficient of friction between the blocks,
$\mu = 0.2$
Let a_1 and a_2 be accelerations of upper and lower blocks respectively. Then
the equation of motion of upper block is



$F - f = m_1 a_1$

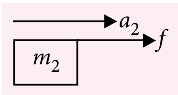
$$a_1 = \frac{F - f}{m_1} = \frac{F - \mu m_1 g}{m_1}$$

$$= \frac{20\text{ N} - (0.2)(3\text{ kg})(10\text{ m s}^{-2})}{3\text{ kg}}$$

$$= \frac{20\text{ N} - 6\text{ N}}{3\text{ kg}} = \frac{14}{3}\text{ m s}^{-2}$$

The equation of motion of lower block is

$f = m_2 a_2$



$$a_2 = \frac{f}{m_2} = \frac{\mu m_1 g}{m_2}$$

$$= \frac{(0.2)(3\text{ kg})(10\text{ m s}^{-2})}{10\text{ kg}} = \frac{6\text{ N}}{10\text{ kg}} = 0.6\text{ m s}^{-2}$$

27. (d) : The filament resistance at 0°C is

$R_0 = 4\ \Omega$ (given)

and at 425°C is

$$R_{425} = \frac{3.5\text{ V}}{0.28\text{ A}} = 12.5\ \Omega$$

Let α be required temperature coefficient of resistance.

As $R_T = R_0(1 + \alpha T)$

$$\therefore\ \alpha = \frac{R_T - R_0}{R_0 T} = \frac{R_{425} - R_0}{R_0(425°\text{C})}$$

$$= \frac{12.5\ \Omega - 4\ \Omega}{4\ \Omega(425°\text{C})} = 5\times 10^{-3}\ °\text{C}^{-1}$$

28. (a) : Here, $A_{max} = 10$ V, $\mu = \frac{2}{3}$

Modulation index, $\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

$$\frac{2}{3} = \frac{10\text{ V} - A_{min}}{10\text{ V} + A_{min}}$$

$2(10\text{ V} + A_{min}) = 3(10\text{ V} - A_{min})$

$20\text{ V} + 2A_{min} = 30\text{ V} - 3A_{min}$

$5A_{min} = 10$ V or $A_{min} = 2$ V

29. (c) : The work done in moving a body along a smooth inclined plane is

$W = mg\sin\theta s$

where m is the mass of the body, θ is the angle of inclination, s is the distance moved by the body along the inclined plane.

Here,

$m = 300$ kg, $s = 10$ m, $g = 9.8$ m s^{-2}, $\theta = 30°$

$\therefore\ W = (300\text{ kg})(9.8\text{ m s}^{-2})(10\text{ m})\sin 30°$

$= 14700$ J

30. (d) : The magnitude of electric and magnetic fields in an electromagnetic wave are related as $B_0 = \frac{E_0}{c}$

Thus, $\frac{E_0}{B_0} = c$

where c is the velocity of electromagnetic wave in free space.

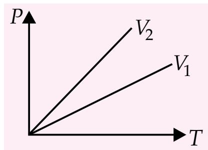
31. (b) : According to gas equation,

$PV = nRT$

$$\therefore\ P = \frac{nR}{V}T$$

Comparing this with $y = mx$, P-T graph is a straight line passing through origin with slope $= \frac{nR}{V}$.

Slope $\propto \frac{1}{V}$



As $V_1 > V_2$ (given)

$\therefore$ (Slope)$_1$ < (Slope)$_2$

or (Slope)$_2$ > (Slope)$_1$

32. (a) : The magnetisation of a paramagnetic sample is $M = C\frac{B}{T}$

where B is the external magnetic field, T is the absolute temperature and C is the Curie's constant.

$$\therefore\ \frac{M_1}{M_2} = \frac{CB_1/T_1}{CB_2/T_2} = \left(\frac{B_1}{B_2}\right)\left(\frac{T_2}{T_1}\right)$$

$$M_2 = M_1\left(\frac{B_2}{B_1}\right)\left(\frac{T_1}{T_2}\right)$$

$$= (0.8\text{ A m}^{-1})\left(\frac{0.4\text{ T}}{0.8\text{ T}}\right)\left(\frac{5\text{ K}}{20\text{ K}}\right)$$

$= 0.1$ A m^{-1}

33. (b) : Here,

Amplitude of oscillation, $A = 4$ cm $= 4 \times 10^{-2}$ m

Maximum kinetic energy, $K_{max} = 1$ J

As $K_{max} = \frac{1}{2}kA^2$

$\therefore$ The force constant of the spring is

$$k = \frac{2K_{max}}{A^2} = \frac{2(1\text{ J})}{(4\times 10^{-2}\text{ m})^2} = 1250\text{ N m}^{-1}$$

34. (b) : Here,

Change in collector current, $\Delta I_C = 2$ mA

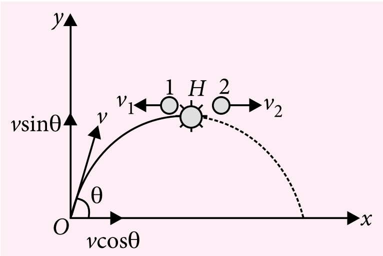
Current amplification factor, $\beta = 50$

As $\beta = \frac{\Delta I_C}{\Delta I_B}$

$$\therefore \quad \Delta I_B = \frac{\Delta I_C}{\beta} = \frac{2\text{ mA}}{50} = 0.04\text{ mA} = 0.04 \times 10^{-3}\text{ A}$$

$$= 40 \times 10^{-6}\text{ A} = 40\ \mu\text{A}$$

35. (c) : The flight of the shell before explosion is shown in figure where H is the highest point.



Velocity of the shell at $H = v\cos\theta$

At H, it explodes into two pieces each of mass $m/2$ and denoted by 1 and 2 respectively. Let v_1 and v_2 be velocities of the pieces 1 and 2 immediately after the explosion.

As the piece 1 retraces its path to the cannon,

$\therefore \quad v_1 = -v\cos\theta$

Applying the law of conservation of momentum,

$$mv\cos\theta = \frac{m}{2}v_1 + \frac{m}{2}v_2$$

$$mv\cos\theta = \frac{m}{2}(-v\cos\theta) + \frac{m}{2}v_2$$

$$v_2 = 3v\cos\theta$$

36. (c) : Here,

Length of the rod, $l = 2$ m

Angular velocity, $\omega = 100$ rad s^{-1}

Magnetic field, $B = 0.3$ T

The potential difference between the ends of the rod is

$$\varepsilon = \frac{1}{2}Bl^2\omega = \frac{1}{2}(0.3\text{ T})(2\text{ m})^2(100\text{ rad s}^{-1})$$

$$= 60\text{ V}$$

37. (c) : Here,

Displacement, $\Delta\vec{r} = (5\hat{i} + 3\hat{j} - 4\hat{k})$ m

Force, $\vec{F} = (6\hat{i} + 6\hat{j} + 4\hat{k})$ N

$$\text{Velocity, } \vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(5\hat{i} + 3\hat{j} - 4\hat{k})\text{ m}}{5\text{ s}}$$

$$= \left(\hat{i} + \frac{3}{5}\hat{j} - \frac{4}{5}\hat{k}\right)\text{m s}^{-1}$$

Power, $P = \vec{F}\cdot\vec{v}$

$$= (6\hat{i} + 6\hat{j} + 4\hat{k})\text{ N}\cdot\left(\hat{i} + \frac{3}{5}\hat{j} - \frac{4}{5}\hat{k}\right)\text{m s}^{-1}$$

$$= 6 + \frac{18}{5} - \frac{16}{5}\text{ W}$$

$$= \frac{30 + 18 - 16}{5}\text{ W} = \frac{32}{5}\text{ W} = 6.4\text{ W}$$

38. (d) : The given transition $n = 2$ to $n = 1$ corresponds to first line of Lyman series.

For Lyman series,

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \text{ where } n = 2, 3, 4, ...$$

For first line, $n = 2$

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R \qquad \text{... (i)}$$

For minimum wavelength of the series, $n = \infty$

$$\therefore \quad \frac{1}{\lambda_{min}} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R \qquad \text{... (ii)}$$

Dividing eqn. (i) by eqn. (ii), we get

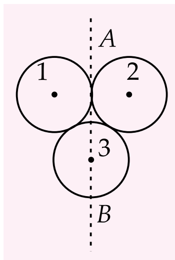
$$\frac{\lambda_{min}}{\lambda} = \frac{3}{4} \quad \text{or} \quad \lambda_{min} = \frac{3}{4}\lambda$$

But $\lambda = 122$ nm (given)

$$\therefore \quad \lambda_{min} = \frac{3}{4}(122\text{ nm}) = 91.5\text{ nm} = 915\text{ Å}$$

39. (b) : Moment of inertia of each disc of mass M and radius R about its diameter is

$$I_{dia} = \frac{1}{4}MR^2$$



Moment of inertia of disc 1 about the axis AB is

$I_1 = I_{dia} + MR^2$

(by theorem of parallel axes)

$$= \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

Similarly, $I_2 = \frac{5}{4}MR^2$

Moment of inertia of disc 3 about the axis AB is

$$I_3 = I_{dia} = \frac{1}{4}MR^2$$

$\therefore$ Moment of inertia of the system about the axis AB is $I = I_1 + I_2 + I_3$

$$= \frac{5}{4}MR^2 + \frac{5}{4}MR^2 + \frac{1}{4}MR^2$$

$$= \frac{11}{4}MR^2$$

40. (a) : Number of waves in glass slab is

$$N_1 = \frac{\text{Thickness of glass slab}}{\text{Wavelength of light in glass}} = \frac{8\text{ cm}}{\lambda_{glass}}$$

Number of waves in water is

$$N_2 = \frac{\text{Length of path of water}}{\text{Wavelength of light in water}} = \frac{10\text{ cm}}{\lambda_{water}}$$

As $N_1 = N_2$ (given)

$$\therefore \quad \frac{8\text{ cm}}{\lambda_{glass}} = \frac{10\text{ cm}}{\lambda_{water}}$$

$$\text{or} \quad \frac{\lambda_{glass}}{\lambda_{water}} = \frac{8\text{ cm}}{10\text{ cm}} = \frac{4}{5} \qquad \text{...(i)}$$

As refractive index of a medium,

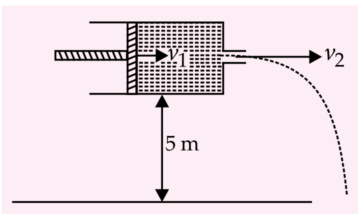
$$\mu = \frac{\text{Wavelength of light in vacuum}}{\text{Wavelength of light in medium}}$$

$$\therefore \quad \frac{\mu_{glass}}{\mu_{water}} = \frac{\lambda_{water}}{\lambda_{glass}}$$

$$\text{or} \quad \mu_{glass} = \frac{\lambda_{water}}{\lambda_{glass}}\mu_{water}$$

$$= \left(\frac{5}{4}\right)\left(\frac{4}{3}\right) = \frac{5}{3} \qquad \text{(using (i))}$$

41. (c) :



Here,

Diameter of syringe, $d_1 = 1\text{ cm} = 10^{-2}\text{ m}$

Diameter of nozzle, $d_2 = 1\text{ mm} = 10^{-3}\text{ m}$

According to equation of continuity, $A_1v_1 = A_2v_2$

$$\pi\left(\frac{d_1}{2}\right)^2 v_1 = \pi\left(\frac{d_2}{2}\right)^2 v_2$$

$$\pi\left(\frac{10^{-2}\text{ m}}{2}\right)^2 (0.5\text{ m s}^{-1}) = \pi\left(\frac{10^{-3}\text{ m}}{2}\right)^2 v_2$$

$$\text{or} \quad v_2 = \left(\frac{10^{-4}\text{m}^2}{10^{-6}\text{m}^2}\right)(0.5\text{ m s}^{-1})$$

$$= 50\text{ m s}^{-1}$$

Horizontal distance travelled by the liquid jet is

$$R = v_2\sqrt{\frac{2h}{g}} = (50\text{ m s}^{-1})\sqrt{\frac{2\,(5\text{ m})}{10\text{ m s}^{-2}}} = 50\text{ m}$$

42. (d) : Number of atoms present in a radioactive substance is $N_0 = \left(\frac{m}{M}\right)N_A$

where m is the mass of the substance, M its molecular weight and N_A is Avogadro number.

As $m = V\rho$

$$\therefore \quad N_0 = \left(\frac{V\rho}{M}\right)N_A \qquad \text{...(i)}$$

The activity of the substance after time t is

$$R = \lambda N = \lambda N_0 e^{-\lambda t} \qquad (\because\ N = N_0e^{-\lambda t})$$

$$R = \lambda\left(\frac{V\rho}{M}\right)N_A e^{-\lambda t} \qquad \text{(using (i))}$$

$$R = \left(\frac{\lambda V\rho N_A}{M}\right)e^{-\lambda t}$$

43. (a) : If I_0 is the intensity of unpolarised light, then intensity of light transmitted through first polariser is $I_1 = \frac{I_0}{2}$

Intensity of light transmitted through second polariser is

$$I_2 = I_1\cos^2\theta$$

(where θ is the angle between axes of two polarisers)

$$= \frac{I_0}{2}\cos^2 45° = \frac{I_0}{2}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{4}$$

44. (a) : The magnitude of electric field intensity due to an infinitely long straight wire of uniform linear charge density λ is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

where r is the perpendicular distance of the point from the wire.

Here, $\lambda = \frac{1}{3}\text{ C m}^{-1}$, $r = 18\text{ cm} = 18 \times 10^{-2}\text{ m}$

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\text{ N m}^2\text{ C}^{-2}$$

$$\therefore \quad E = \frac{(9\times10^9\text{ N m}^2\text{C}^{-2})(2)\left(\frac{1}{3}\text{C m}^{-1}\right)}{(18\times10^{-2}\text{ m})}$$

$$= \frac{1}{3}\times10^{11}\text{N C}^{-1} = 0.33\times10^{11}\text{ N C}^{-1}$$

45. (d) : The thermal conductivity of a rod depends on material of the rod.

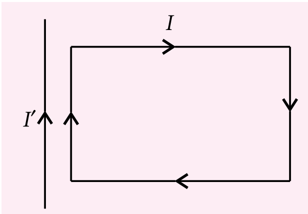
EXAM ON
14th to 29th MAY

BITSAT
PRACTICE PAPER

1. A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is

(a) blue (b) green
(c) violet (d) red

2. A rectangular loop carrying a current I is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If steady current I' is established in the wire as shown in the figure, the loop will



(a) rotate about an axis parallel to the wire
(b) move away from the wire
(c) move towards the wire
(d) remain stationary.

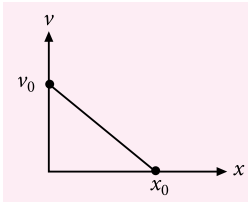
3. Force between two identical charges placed at a distance of r in a vacuum is F. Now a slab of dielectric constant 4 is inserted between these two charges. If the thickness of the slab is $r/2$, then the force between the charges will become

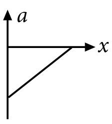
(a) F (b) $\frac{3}{5}F$
(c) $\frac{4}{9}F$ (d) $\frac{F}{4}$

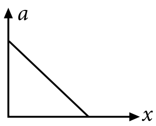
4. In an ideal double-slit experiment, when a glass plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is

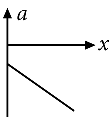
(a) 2λ (b) $2\lambda/3$
(c) $\lambda/3$ (d) λ

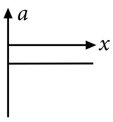
5. The velocity displacement graph of a particle moving along a straight line is shown. The most suitable acceleration-displacement graph will be



(a) 

(b) 

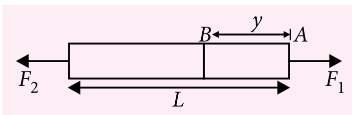
(c) 

(d) 

6. A thin circular ring of area A is held perpendicular to a uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is

(a) $\frac{BR}{A}$ (b) $\frac{AB}{R}$ (c) ABR (d) $\frac{B^2A}{R^2}$.

7. What is the tension in a rod of length L and mass M at a distance y from F_1, when the rod is acted on by two unequal forces F_1 and F_2 ($<F_1$) as shown in figure.



(a) F_1 (b) $F_1 - F_2$
(c) $F_1\left(\frac{y}{L}\right)+F_2\left(1-\frac{y}{L}\right)$ (d) $F_1\left(1-\frac{y}{L}\right)+F_2\left(\frac{y}{L}\right)$

8. A thin convex lens of focal length 25 cm is cut into two pieces, 0.5 cm above the principal axis. The top part is placed at (0, 0) and an object placed at

(–50 cm, 0). The coordinates of the image will be
(a) (50 cm, –0.5 cm) (b) (–50 cm, –1 cm)
(c) (50 cm, –1 cm) (d) (–50 cm, –0.5 cm)

9. Imagine a light planet revolving around a massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force of attraction between planet and the star is proportional to $r^{-5/2}$, then the relation between T and r will be
(a) $T \propto r^{-2}$ (b) $T^2 \propto r^{7/2}$
(c) $T^3 \propto r^4$ (d) $T \propto r^{3/2}$

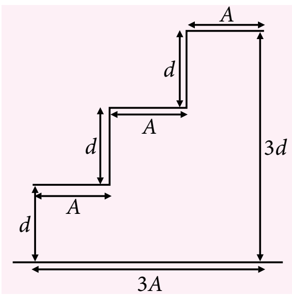
10. A 4 μF capacitor and a resistance of 2.5 MΩ are in series with 12 V battery. The time after which the potential difference across the capacitor is 3 times the potential difference across the resistor is
[Given, ln2 = 0.693]
(a) 13.86 s (b) 6.93 s (c) 7 s (d) 14 s.

11. A boat which has a speed of 5 km h^{-1} in still water crosses a river of width 1 km along the shortest possible path in 15 minute. The velocity of the river water in km h^{-1} is
(a) 1 (b) 3 (c) 4 (d) $\sqrt{41}$

12. A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities constant along the length of the conductor is/are
(a) current, electric field and drift speed
(b) drift speed only
(c) current and drift speed
(d) current only.

13. The number of particles given by $n = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$ are crossing a unit area perpendicular to x-axis in unit time, n_1, n_2 are number of particles per unit volume, for the values of x meant to be x_1 and x_2. What is the dimensional formula of diffusion constant D?
(a) [L] (b) $[L^2T^{-2}]$
(c) $[L^2T^{-1}]$ (d) $[L^2T]$

14. The expression for the equivalent capacitance of the system shown in figure is (A is the corss-sectional area of one of the plates)



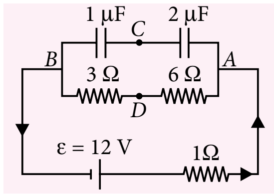
(a) $\varepsilon_0 A/3d$
(b) $\frac{3\varepsilon_0 A}{d}$
(c) $\varepsilon_0 A/6d$ (d) $\frac{11\varepsilon_0 A}{6d}$

15. What is the power of an engine, which can just pull a train of mass 5000 quintals up an incline of 1 in 50 at the rate of 54 km h^{-1}. The resistance due to friction is 0.8 N/quintal. Take g = 9.8 m s^{-2}.
(a) 1530 kW (b) 1470 kW
(c) 3060 kW (d) 5000 kW

16. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. The work function in eV could be
(a) 1.04 (b) 2.08
(c) 1.40 (d) 1.68

17. If the shear modulus of a wire material is 5.9×10^{11} dyne/cm^2 then the potential energy of a wire of 4×10^{-3} cm in diameter and 5 cm long twisted through an angle of 10′, is
(a) 1.253×10^{-12} J (b) 2×10^{-12} J
(c) 1.00×10^{-12} J (d) 0.8×10^{-12} J

18. In given figure, the potential difference between the points C and D is



(a) 7.2 V (b) 3.6 V
(c) 10.8 V (d) 0 V

19. A point mass is subjected to two simultaneous sinusoidal displacements in X-direction :
$x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$.
Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are
(a) $\sqrt{2}A, \frac{3\pi}{4}$ (b) $A, \frac{4\pi}{3}$
(c) $\sqrt{3}A, \frac{5\pi}{6}$ (d) $A, \frac{\pi}{3}$

20. A 100 V ac source of frequency 500 Hz is connected to an LCR circuit with L = 8.1 mH, C = 12.5 μF and R = 10 Ω, all connected in series. The potential difference across the resistance is
(a) 25 V (b) 50 V
(c) 75 V (d) 100 V

21. A particle is projected upward from the surface of the earth (radius R) with a kinetic energy equal to half the minimum value needed for it to escape. To which height does it rise above the surface of the earth?
(a) $2R$ (b) R (c) $3R/2$ (d) $R/2$

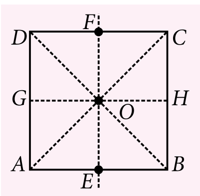
22. If K_α-radiation of Mo ($Z = 42$) has a wavelength 0.71 Å, the wavelength of the corresponding radiation of Cu($Z = 29$) is equal to
(a) 1.52 Å (b) 0.71 Å
(c) 0.36 Å (d) 2.5 Å

23. In the test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m s^{-1} and 63 m s^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2? Take the density of air to be 1.3 kg m^{-3}.
(a) 10^3 N (b) 10^5 N
(c) 1.5×10^3 N (d) 3×10^3 N

24. Two particles have charges $+ 4.0 \times 10^{-5}$ C and -4.0×10^{-5} C and each has a mass of 5 g. When they are 1.0 m apart, they are released from rest. The speed of each particle when their separation becomes 50 cm would be
(a) 26 m s^{-1} (b) 120 m s^{-1}
(c) 53.7 m s^{-1} (d) 82.7 m s^{-1}

25. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases at a temperature of 27°C and pressure 1×10^5 N m^{-2}. The total mass of the mixture is 28 g. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the mass of the individual gases in the container, assuming them to be ideal. (Universal gas constant, R = 8.314 J mol^{-1} K^{-1})
(a) 4 g, 24 g (b) 20 g, 8 g
(c) 26 g, 2 g (d) 6 g, 22 g

26. For the given uniform square lamina $ABCD$, whose centre is O,



(a) $I_{AC} = \sqrt{2} I_{EF}$
(b) $\sqrt{2} I_{AC} = I_{EF}$
(c) $I_{AD} = 3I_{EF}$
(d) $I_{AC} = I_{EF}$

27. One gram of radium is reduced by 2 mg in 5 years by α-decay. The half-life period of radium is
(a) 300 years (b) 585 years
(c) 1735 years (d) 2105 years

28. A small sphere of mass 0.2 g hangs by a thread between two parallel vertical plates 5 cm apart. The charge on the sphere is 6×10^{-9} C. What is the potential difference between the plates if the thread assumes an angles of 30° with the vertical?
(a) 9.4×10^3 V (b) 9.4×10^4 V
(c) 5.2×10^3 V (d) 5.2×10^4 V

29. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively
(a) g, g (b) $a, g - a$
(c) $g - a, g$ (d) a, g

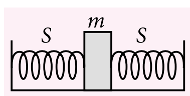
30. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre is
(a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$
(c) $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$ (d) $\frac{\mu_0 IN}{2(b-a)} \ln\left(\frac{a}{b}\right)$

31. The opposite faces of a cubical block of iron of cross-section 4 cm^2 are kept in contact with steam and melting ice. The quantity of ice melted at the end of 10 min would be
(Given, K for iron = 0.2 cal s^{-1} $cm^{-1}C^{\circ -1}$).
(a) 0.3 kg (b) 0.5 kg
(c) 1.0 kg (d) 1.5 kg

32. A sphere of mass m moving with a velocity u hits another stationary sphere of same mass. If e is the coefficient of restitution, what is the ratio of the velocities of two spheres after the collision?
(a) 1 (b) e
(c) $\frac{1+e}{1-e}$ (d) $\frac{2-e}{1+e}$

33. The output of a step-down transformer is measured to be 24 V when connected to a 12 W light bulb. The value of the peak current is
(a) $(1/\sqrt{2})$ A (b) $\sqrt{2}$ A
(c) 2 A (d) $(2\sqrt{2})$ A

34. When a mass m is connected individually to two springs S_1 and S_2, the oscillation frequencies are υ_1 and υ_2 respectively. If the same



mass is attached to the two springs as shown in the figure, the oscillation frequency would be

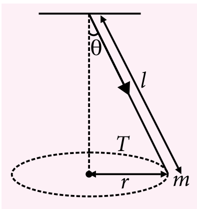
(a) $\upsilon_1 + \upsilon_2$ (b) $\sqrt{\upsilon_1^2 + \upsilon_2^2}$

(c) $\left(\frac{1}{\upsilon_1} + \frac{1}{\upsilon_2}\right)^{-1}$ (d) $\sqrt{\upsilon_1^2 - \upsilon_2^2}$

35. Mercury has an angle of contact equal to 140° with soda-lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 N m^{-1}. Density of mercury is 13.6 × 10^3 kg m^{-3} (cos140° = – 0.7660)

(a) 2.9 mm (b) 7.2 mm
(c) 3.5 mm (d) 5.3 mm

36. A string of length l fixed at one end carries a mass m at the other end. The string makes $\frac{2}{\pi}$ rev s^{-1} around the horizontal axis through the fixed end as shown in the figure. The tension in the string is



(a) 16 ml (b) 4 ml
(c) 8 ml (d) 2 ml

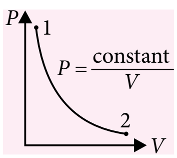
37. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v_s m s^{-1}. The velocity of sound in air is 300 m s^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, the maximum value of v_s upto which he can hear the whistle is

(a) $15/\sqrt{2}$ m s^{-1} (b) 15 m s^{-1}
(c) 30 m s^{-1} (d) $15\sqrt{2}$ m s^{-1}

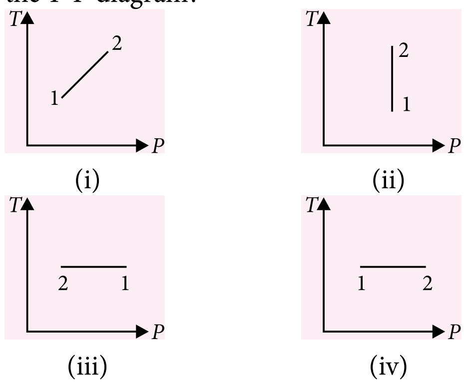
38. A sinusoidal voltage is applied directly across an 8.00 μF capacitor. The frequency of the source is 3.00 kHz and the voltage amplitude is 30.0 V. The displacement current between the plates of the capacitor is

(a) 1.9 A (b) 2.5 A
(c) 4.5 A (d) 5.2 A

39. Consider P-V diagram for an ideal gas shown in figure.



Out of the following diagrams, which represents the T-P diagram?



(a) (iv) (b) (ii)
(c) (iii) (d) (i)

40. A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0.1 m. If the height of the inclined plane is 4 m, what is its rotational K.E., when it reaches the foot of the plane?

(a) 26 J (b) 13 J (c) 52 J (d) 78 J

SOLUTION

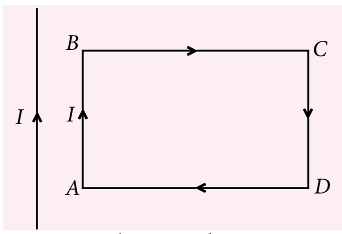
1. (d): Since $\lambda_r > \lambda_b > \lambda_v$ and $\mu = a + b/\lambda^2$

So, $\mu_r < \mu_g < \mu_b < \mu_v$

or $v_r > v_g > v_b > v_b$ ($\because v = c/\mu$),

So, red colour (with largest velocity in glass slab) is the first to emerge.

2. (c) : Current I' in wire and current I in AB are parallel and in same direction.



AB will be attracted towards wire. Wire and CD carry parallel currents in opposite directions. Repulsion occurs between them. The distance is more from wire and so the force of repulsion on CD will be less than the force of attraction on AB.

BC and DA are at right angles to wire and carry currents in opposite directions. Their net force will be zero.

Due to greater attractive force on AB, the loop will move towards the wire.

3. (c) : In vacuum, $F = \frac{q^2}{4\pi\varepsilon_0 r^2}$

With the dielectric in between the charges,

BRAIN MAP

ELECTROSTATICS

In dry weather, a spark is produced by walking across certain types of carpet and then bringing one of the fingers near a metal doorknob, or even a person. Multiple sparks can be produced when a sweater or cloth is pulled from the body. It reveals that electric charge is present in our bodies, sweaters, carpets, doorknobs, computers etc. In fact, every object contains a vast amount of electric charge.

Electric Charge

- A physical quantity responsible for electromagnetism. There are only two kinds of charge, positive and negative.
- **Conservation of charge :** The total charge of an isolated system is always conserved.
- **Quantisation of charge :** $Q = ne$
  where, $n = \pm 1, \pm 2, \pm 3, ...$ and $e = 1.6 \times 10^{-19}$ C

Coulomb's Law

- Electric force between two point charges :
  $$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} = -\vec{F}_{21}$$
  where $K = 1$ for free space, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2}
- Coulomb's law agrees with Newton's third law.
- Force on charge q_1 due to remaining charges $q_2, q_3, q_4, ... q_n$ in the region.
  $$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{r}_{1i}$$
  It is a vector sum of all the forces acting on point charge q_1.

Electric Field

- Due to a point charge, $\vec{E} = \frac{\vec{F}}{q_{\text{test}}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$
- Due to positive charge, electric field is away from the charge and due to negative charge, it is towards the charge.
- Due to charge distribution,

  Linear charge, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum\frac{\lambda\Delta L}{r^2}\hat{r}$

  Surface charge, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum\frac{\sigma\Delta S}{r^2}\hat{r}$

  Volume charge, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum\frac{\rho\Delta V}{r^2}\hat{r}$

Electric Flux and Gauss's Law

- Electric flux through a plane surface area S held in a uniform electric field
  $$\Delta\phi_E = \vec{E}\cdot\Delta\vec{S} = E\Delta S\cos\theta$$
- According to Gauss's law, electric flux through a closed surface S is equal to $(1/\varepsilon_0)$ times charge enclosed.
  $$\phi_E = \oint\vec{E}\cdot d\vec{S} = \frac{q_{en}}{\varepsilon_0}$$

Applications of Gauss's Law

- Electric field due to a long straight wire of uniform linear charge density $E = \frac{\lambda}{2\pi\varepsilon_0 r}$
- Electric field due to an infinite plane sheet of uniform surface charge density σ, $E = \frac{\sigma}{2\varepsilon_0}$
- Electric field due to two positively charged parallel plates with charge densities σ_1 and σ_2 such that $\sigma_1 > \sigma_2 > 0$,

  $E = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)$ (Outside the plates)

  $E = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)$ (Inside the plates)
- Electric field due to two equally and oppositely charged parallel plates,
  $E = 0$ (For outside points), $E = \frac{\sigma}{\varepsilon_0}$ (For inside points)
- Electric field due to a thin spherical shell of charge density σ and radius R,

  $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ For $r > R$ (Outside points)

  $E = 0$ For $r < R$ (Inside points)

  $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ For $r = R$ (At the surface)

  where, $q = 4\pi R^2\sigma$.
- Electric field of a solid sphere of uniform charge density ρ and radius R

  $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ For $r > R$ (Outside points)

  $E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$ For $r < R$ (Inside points)

  $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ For $r = R$ (At the surface)

  where $q = \frac{4}{3}\pi R^3\rho$

Electric Potential

- Work done by an external force (equal and opposite to the electric force) in bringing a unit positive charge from infinity to a point = electrostatic potential (V) at that point.
- Due to a point charge : $V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
- Potential difference between two points,
  $$V_P - V_R = \frac{U_P - U_R}{q} = \frac{W_{RP}}{q}$$
- Potential at a point P, due to system of charges $q_1, q_2, q_3, ...$
  $$V_P = V_1 + V_2 + V_3 + ... = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ...\right)$$
  It is algebraic sum of the potentials due to the individual charges.

Relation between Field and Potential

- $E = -\frac{dV}{dr}$
- Electric field is in the direction in which the potential decreases steepest.
- Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.

Electrostatics of Conductors

- At the surface of a charged conductor, electrostatic field must be normal to the surface at every point.
- The interior of a conductor can have no excess charge in the static situation.
- Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.
- Electric field at the surface of a charged conductor $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

Potential of Charge Distributions

Charge distribution	Formula
Uniformly charged spherical shell	$V_i = V_s = \frac{Kq}{R} = \frac{\sigma R}{\varepsilon_0}$, $V_o = \frac{Kq}{r}$
Uniformly charged solid sphere	$V_i = \frac{Kq}{R^3}(1.5R^2 - 0.5r^2)$ $V_s = \frac{Kq}{R}$, $V_o = \frac{Kq}{r}$
On the axis of uniformly charged ring	$V = \frac{Kq}{\sqrt{R^2 + x^2}}$ At centre, $x = 0$ ∴ $V = \frac{Kq}{R}$
Infinitely long line charge	$PD = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right)$

Electric Potential Energy

- Electric potential energy of a system of two point charges, $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$
- Electric potential energy of a system of N point charges, $U = \frac{1}{4\pi\varepsilon_0}\sum_{\substack{\text{all pairs}\\(j>k)}}\frac{q_j q_k}{r_{jk}}$

Electric Dipole

- A pair of equal and opposite charges q and $-q$ separated by a small distance $2a$.
- The direction of dipole is the direction from $-q$ to q.
- Dipole moment, $p = q \times 2a$
- Electric Potential, $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$
- Dipole field at an axial point at distance r from the centre of the dipole $E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2 - a^2)^2}$
  when $r >> a$ $E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$
- Dipole field at an equatorial point at distance r from the centre of the dipole is $E_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2 + a^2)^{3/2}}$
  when $r >> a$ $E_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$
- Torque, $\tau = pE\sin\theta$
- Potential energy of an electric dipole in a uniform electric field, $U = -pE(\cos\theta_2 - \cos\theta_1)$
- ➢ If initially the dipole is perpendicular to the field E, $\theta_1 = 90°$ and $\theta_2 = \theta$, then $U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$
- ➢ If initially the dipole is parallel to the field E, $\theta_1 = 0°$ and $\theta_2 = \theta$, then $U = -pE(\cos\theta - 1) = pE(1 - \cos\theta)$

Dielectric and Polarisation

- A dielectric develops a net dipole moment in an external electric field.
- Polarisation $= \frac{\text{net dipole moment}}{\text{volume}}$
  For a linear isotropic dielectric, $\vec{P} = \chi_e\vec{E}$
- Surface charge density due to polarisation,
  $$\sigma_P = \vec{P}\cdot\hat{n} = P\cos\theta$$

Capacitors and Capacitance

- A capacitor is a system of two conductors separated by an insulator.
- Capacitance, $C = \frac{q}{V_o} = \frac{q}{V_1 - V_2}$
- For a spherical conductor, $C = 4\pi\varepsilon_0 R$
- Energy stored in a charged conductor or capacitor,
  $$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

Different Types of Capacitors

- Capacity of parallel plate capacitor, $C = \frac{K\varepsilon_0 A}{d}$
- Induced charges, $\sigma_i = \sigma\left(1 - \frac{1}{K}\right)$ or $q_i = q\left(1 - \frac{1}{K}\right)$
- Capacity of capacitor partially filled with a dielectric, $C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$
- Force of attraction between plates of capacitors, $F = \frac{q^2}{2A\varepsilon_0}$
- Capacity of spherical capacitor, $C = 4\pi\varepsilon_0\left(\frac{ab}{b - a}\right)$
- Capacity of cylindrical capacitor per unit length, $\frac{2\pi\varepsilon_0}{\ln(b/a)}$

Combination of Capacitors

- Capacitors in series : $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... = \sum_i\frac{1}{C_i}$
- Capacitors in parallel : $C = C_1 + C_2 + C_3 + ... = \sum_i C_i$

Van de Graaff Generator

- By means of a moving belt and suitable brushes charge is continuously transferred to the shell and potential difference of the order of several million volts is built up, which can be used for accelerating charged particles.

$$F' = \frac{q^2}{4\pi\varepsilon_0 K r'^2}$$

If $F' = F$, then $Kr'^2 = r^2$ or $r = \sqrt{K}\, r'$.

In the given situation,

$$F' = \frac{q^2}{4\pi\varepsilon_0\left[\frac{r}{2}+\sqrt{4}\frac{r}{2}\right]^2} = \frac{4}{9}\frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{4}{9}F$$

4. (a) : When glass is introduced in the path of one of interfering beams then an extra path difference is produced. Intensity of the position of central maxima occured previously remains unchanged if extra path difference is equal to integral multiple of wavelength of light *i.e.*, $\Delta x = (\mu - 1)t = n\lambda$, $n = 1, 2, 3$

for minimum thickness, $n = 1$, $t = \frac{n\lambda}{(\mu-1)} = \frac{1\lambda}{(1.5-1)}$

$t = 2\lambda$

5. (a) : The velocity-displacement graph is given. Corresponding acceleration-displacement graph is required.

From v-x graph, the corresponding equation is

$$v = \left(-\frac{v_0}{x_0}\right)x + v_0 \qquad \text{(Pattern, } y = mx + c\text{)}$$

$$\therefore \quad \frac{dv}{dt} = \left(-\frac{v_0}{x_0}\right)\frac{dx}{dt} \quad \text{or} \quad a = \left(-\frac{v_0}{x_0}\right)v$$

$$\text{or} \quad a = \left(-\frac{v_0}{x_0}\right) \times \left[\left(-\frac{v_0}{x_0}\right)x + v_0\right]$$

$$\text{or} \quad a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0} \qquad \text{(Pattern, } y = mx + c\text{)}$$

Slope of the line $= \left(\frac{v_0}{x_0}\right)^2$

Slope is positive. Therefore θ is acute.

Intercept $= \frac{v_0^2}{x_0}$. It is negative.

The slope and intercept reveal that the graph (a) is the most suitable representation.

6. (b) : EMF induced in the ring $= \varepsilon$

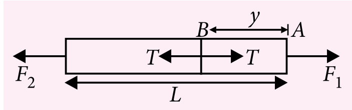
$$\therefore \quad \varepsilon = \frac{-d\phi}{dt}, \quad \text{where } \phi = \text{flux} = BA$$

$$\text{or} \quad IR = \frac{-d\phi}{dt} \qquad \text{or} \quad RI\,dt = -\,d\phi$$

$$\text{or} \quad R\int I dt = -\int d\phi \qquad \text{or} \quad Rq = |\,\phi\,|$$

$$\text{or} \quad q = \frac{BA}{R}$$

7. (d) : Here, as is clear in figure, net force on the rod $F = (F_1 - F_2)$



$\therefore$ Acceleration of rod, along F_1

$$a = \frac{F}{M} = \frac{(F_1 - F_2)}{M} \qquad \text{...(i)}$$

Mass of part AB of the rod $= \left(\frac{M}{L}\right)y$

Let T be the tension in the rod at B.

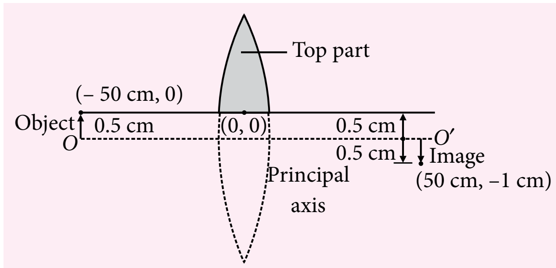
Equation of motion of part AB of the rod is

$$F_1 - T = ma = \frac{M}{L}y\cdot\left(\frac{F_1 - F_2}{M}\right)$$

$$F_1 - T = (F_1 - F_2)\frac{y}{L} \quad \text{or} \quad T = F_1 - (F_1 - F_2)\frac{y}{L}$$

$$T = F_1(1 - y/L) + F_2(y/L)$$

8. (c) : In figure OO' be the principal axis of the convex lens before it is cut into two pieces. In this case, the object is 0.5 cm above OO'.



As $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, $\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{-50} + \frac{1}{25} = \frac{1}{50}$

or $v = 50$ cm

Further, magnification of the image, *i.e.*,

$$m = \frac{v}{u} = \frac{50}{-50} = -1$$

Therefore, the image of the object is formed at 50 cm from the pole and 0.5 cm below the principal axis. Thus, the co-ordinates of the image with respect to time the X-axis through the edge of the top part of the cut lens are (50 cm, –1 cm).

9. (b) : Force of gravitation on the planet
= Centripetal force

$$kr^{-5/2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

$$\text{or} \quad T^2 = \frac{4\pi^2 mr}{kr^{-5/2}} = \frac{4\pi^2 m}{k}\cdot r^{7/2}$$

$$\therefore \quad T^2 \propto r^{7/2}.$$

10. (a) : Charging current $= I = \frac{V_0}{R} e^{-\frac{t}{RC}}$

$\therefore$ Potential difference across $R, V_R = V_0\, e^{-\frac{t}{RC}}$

$\therefore$ Potential difference across $C, V_C = V_0 - V_0 e^{-t/RC}$

According to question, $V_C = 3V_R$

or $1 - e^{-t/RC} = 3\, e^{-t/RC}$ or $1 = 4\, e^{-t/RC}$

or $\ln 1 = \ln 4 + \left(-\frac{t}{RC}\right)$

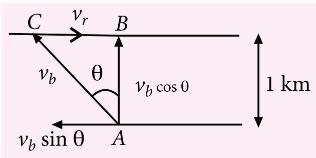
or $t = RC \times 2\ln 2$

$t = 2.5 \times 10^6 \times 4 \times 10^{-6} \times 2 \times 0.693$

or $t = 13.86$ sec.

11. (b) : Time $t = 15$ min $= \frac{15}{60} = \frac{1}{4}$ hour

velocity $= \frac{\text{distance}}{\text{time}}$



$v_b \cos\theta = \frac{1}{1/4} = 4 \frac{\text{km}}{\text{h}}$

or $5 \cos\theta = 4$

or $\cos\theta = \frac{4}{5}$ $\therefore \sin\theta = \frac{3}{5}$

Consider the triangle of velocity ABC,

$\frac{v_r}{v_b} = \frac{3}{5}$ or $\frac{v_r}{5} = \frac{3}{5}$ or $v_r = 3$ km h^{-1}.

12. (d) : $I = neAv_d$ where

v_d = drift speed, A = cross-sectional area.

As A is different at different sections, since the conductor is non-uniform, v_d is different.

Again electric field $E = \frac{I}{\sigma A}$.

E does not remain constant as A varies from section to section of a non-uniform conductor.

Hence current is the only quantity that remains constant along the length of the conductor.

13. (c) : From, $n = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$

$$D = \frac{n(x_2 - x_1)}{-(n_2 - n_1)}$$

Dimensions of n = number/area/time

$= [L^{-2}T^{-1}]$

Dimensions of n_1, n_2 = number/volume $= [L^{-3}]$

Dimensions of $x_2 - x_1$ = difference between two positions = [L]

$\therefore$ $D = \frac{[L^{-2}T^{-1}][L]}{[L^{-3}]} = [L^2T^{-1}]$

14. (d) : Here, $C_1 = \frac{\varepsilon_0 A}{d}$, $C_2 = \frac{\varepsilon_0 A}{2d}$, $C_3 = \frac{\varepsilon_0 A}{3d}$

As the three condensers are in parallel in figure.

$\therefore$ $C_p = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 A}{3d}$

$C_p = \frac{\varepsilon_0 A}{d}\left[1 + \frac{1}{2} + \frac{1}{3}\right] = \frac{11\varepsilon_0 A}{6d}$

15. (a) : Here, power, $P' = ?$, $m = 5000$ quintals

$= 5 \times 10^5$ kg.

$\sin\theta = \frac{1}{50}$, $v = 54$ km h^{-1} $= \frac{54 \times 1000}{60 \times 60}$ m s^{-1} = 15 m s^{-1}

Force of friction, $F = 0.8$ N/quintal

$= 0.8 \times 5000$ N = 4000 N

$P' = \frac{W}{t} = (mg\sin\theta + F) \times \frac{S}{t} = (mg\sin\theta + F) \times v$

$P' = \left(5 \times 10^5 \times 9.8 \times \frac{1}{50} + 4000\right) 15$

$P' = (98000 + 4000) \times 15 = 1530000$ W = 1530 kW

16. (a) : As $K_{max} = h\upsilon - \phi_0 = \frac{hc}{\lambda} - \phi_0$,

According to question, $2K_{max} = K'_{max}$

or $2\left(\frac{hc}{\lambda_1} - \phi_0\right) = \left(\frac{hc}{\lambda_2} - \phi_0\right)$

or $\frac{2hc}{\lambda_1} - 2\phi_0 = \frac{hc}{\lambda_2} - \phi_0$

or $\phi_0 = hc\left[\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right]$

$= (1242 \text{ eV nm})\left[\frac{2}{600 \text{ nm}} - \frac{1}{400 \text{ nm}}\right]$

(as $hc = 1242$ eV nm)

$= (1242 \text{ eV nm})\left(\frac{1}{1200 \text{ nm}}\right) = 1.035$ eV

17. (a) : To twist wire through the angle $d\theta$, it is necessary to do the work

$dW = \tau d\theta$

and $\theta = 10' = \frac{10}{60} \times \frac{\pi}{180} = \frac{\pi}{1080}$ rad

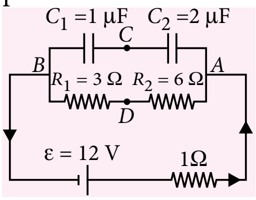
$$W = \int_0^\theta \tau\, d\theta = \int_0^\theta \frac{\eta\pi r^4 \theta\, d\theta}{2l} = \frac{\eta\pi r^4\theta^2}{4l}$$

$$W = \frac{5.9 \times 10^{11} \times 10^{-5} \times \pi (2 \times 10^{-5})^4\, \pi^2}{10^{-4} \times 4 \times 5 \times 10^{-2} \times (1080)^2}$$

$W = 1.253 \times 10^{-12}$ J

18. (b): Since the capacitors are in series, charge on each capacitor is the same. At steady state, the charge on the capacitor is due to terminal potential difference of cell ε.

When steady state is reached, no current flows through the capacitors. The current in the circuit is,



$$I = \frac{12}{1+3+6} = 1.2\text{ A}$$

Terminal potential difference across A and B,

$V = \varepsilon - Ir = 12 - 1.2 \times 1 = 10.8$ V

Capacitors are in series, their effective capacitance

$$C = \frac{1\times 2}{1+2} = \frac{2}{3}\ \mu\text{F}$$

∴ Charge on each capacitor,

$$q = \frac{2}{3} \times 10^{-6} \times 10.8 = 7.2 \times 10^{-6}\text{ C}$$

Charge on capacitor C_2, $q = 7.2 \times 10^{-6}$ C.

Potential difference between A and $C = V_A - V_C =$

$$\frac{q}{C_2} = \frac{7.2\times 10^{-6}}{2\times 10^{-6}} = 3.6\text{ V}$$

Potential difference between A and $D = V_A - V_D$

$= 6 \times 1.2 = 7.2$ V.

Potential difference between C and $D = V_C - V_D$

$= (V_A - V_D) - (V_A - V_C) = 7.2 - 3.6 = 3.6$ V

19. (b): Here, $x_1 = A\sin\omega t$

$$x_2 = A\sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$\therefore \quad x_1 + x_2 = A\sin\omega t + A\sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$= A\sin\omega t + A\left[\sin\omega t\cos\frac{2\pi}{3} + \cos\omega t\sin\frac{2\pi}{3}\right]$$

$$= A\sin\omega t + A\left[\sin\omega t\left(-\frac{1}{2}\right) + \cos\omega t\left(\frac{\sqrt{3}}{2}\right)\right]$$

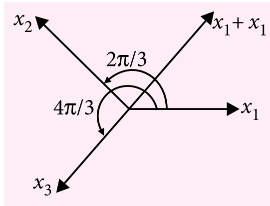
$$= \frac{A}{2}\sin\omega t + \frac{\sqrt{3}}{2}A\cos\omega t$$

$$= A\left[\sin\omega t\cos\frac{\pi}{3} + \cos\omega t\sin\frac{\pi}{3}\right] = A\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$\because \quad x_1 + x_2 + x_3 = 0$$

$$\Rightarrow \quad x_3 = -(x_1 + x_2) = -A\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$= A\sin\left(\omega t + \pi + \frac{\pi}{3}\right)$$



$$x_3 = A\sin\left(\omega t + \frac{4\pi}{3}\right)$$

$$\because \quad x_3 = B\sin(\omega t + \phi)$$

Hence, $B = A, \phi = \dfrac{4\pi}{3}$

20. (d): As $X_L = 2\pi\upsilon L = 2 \times 3.14 \times 500 \times 8.1 \times 10^{-3}$

$$= 25.4\ \Omega,\ X_C = \frac{1}{2\pi\upsilon C}$$

$$= \frac{1}{2\times 3.14\times 500\times 12.5\times 10^{-6}} = 25.4\ \Omega,$$

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\ \Omega$$

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{Z} = \frac{100\text{ V}}{10\ \Omega} = 10\text{ A}$$

$$V_R = I_{\text{rms}}R = (10\text{ A})(10\ \Omega) = 100\text{ V}$$

21. (b): For the particle to escape, K.E. = P.E.

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R} \qquad \text{...(i)}$$

But supplied K.E. $= \dfrac{1}{2}\times\dfrac{1}{2}mv_e^2 = \dfrac{GMm}{2R}$

Suppose the particle rises to a height h, then

$$\frac{1}{2}\times\frac{1}{2}mv_e^2 = \frac{GMm}{R+h}$$

$$\frac{GMm}{2R} = \frac{GMm}{R+h} \qquad \text{(Using (i))}$$

$$\therefore \quad h = R$$

22. (a): From Moseley's law, as $\lambda \propto \dfrac{1}{(Z-1)^2}$

$$\frac{\lambda_{\text{Mo}}}{\lambda_{\text{Cu}}} = \frac{(Z_{\text{cu}} - 1^2)}{(Z_{\text{Mo}} - 1)^2} = \frac{(29-1)^2}{(42-1)^2} = 0.4663$$

or $\lambda_{\text{Cu}} = \dfrac{\lambda_{\text{Mo}}}{0.4663} = \dfrac{0.71\text{ Å}}{0.4663} = 1.52$ Å

23. (c): Here, $v_1 = 70$ m s^{-1}, $v_2 = 63$ m s^{-1}, $A = 2.5$ m^2

For horizontal flow, $P_1 + \dfrac{1}{2}\rho v_1^2 = P_2 + \dfrac{1}{2}\rho v_2^2$

or $(P_2 - P_1) = \dfrac{1}{2}\rho\,(v_1^2 - v_2^2)$

or $(P_2 - P_1) = \dfrac{1}{2}(1.3\text{ kg m}^{-3})[(70\text{ m s}^{-1})^2 - (63\text{ m s}^{-1})^2]$

$= \dfrac{1}{2}(1.3\text{ kg m}^{-3})(931\text{ m}^2\text{ s}^{-2}) = 605\text{ N m}^{-2}$

Lift (force) on the wing, $F = (P_2 - P_1)\,A$

$= (605\text{ N m}^{-2})\,(2.5\text{ m}^2) = 1.5 \times 10^3$ N

24. (c) : As $|\Delta K| = |\Delta U|$, $2\left(\frac{1}{2}mv^2\right) = k\left[\frac{q_1q_2}{r_2} - \frac{q_1q_2}{r_1}\right]$

or $(5\times10^{-3})v^2$

$= (9\times10^9)(4\times10^{-5})(4\times10^{-5})\left(\frac{1}{0.5} - \frac{1}{1}\right) = 14.4$

or $v = \sqrt{\frac{14.4}{5\times10^{-3}}} = 53.7 \text{ m s}^{-1}$

25. (a) : $m_N + m_A = 28$ g ...(i)

Here, m_N and m_A represent masses of neon and argon gases respectively.

$n_N = \frac{m_N}{20}$, $n_A = \frac{m_A}{40}$

As $PV = nRT = (n_N + n_A)RT$

$n_N + n_A = \frac{PV}{RT} = \frac{10^5\times0.02}{8.314\times300} = 0.8$

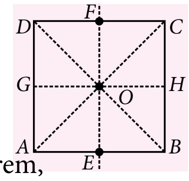
or $\frac{m_N}{20} + \frac{m_A}{40} = 0.8$

or $2m_N + m_A = 32$ ...(ii)

From eqns. (i) and (ii),

$m_N = 4$ g, $m_A = 24$ g

26. (d): Let I be the M.I. of the square lamina about an axis through O and perpendicular to its plane. Then by perpendicular axes theorem,



$I_{AC} + I_{BD} = I$

or $I_{AC} = \frac{I}{2}$ [$\because$ $I_{AC} = I_{BD}$]

Again by perpendicular axes theorem,

$I_{EF} + I_{GH} = I$

or $I_{EF} = \frac{I}{2}$ [$\because$ $I_{EF} = I_{GH}$]

Hence $I_{AC} = I_{EF}$

27. (c) : Given that $N_0 = 1$ g

$N = 1\text{ g} - 2\text{ mg} = 1\text{ g} - 0.002\text{ g} = 0.998\text{ g}$, $t = 5$ y

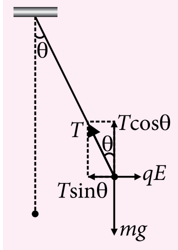
As $N = N_0e^{-\lambda t}$, $\frac{N}{N_0} = e^{-\lambda t}$ or $e^{\lambda t} = N_0/N$

or $e^{5\lambda} = \frac{1\text{g}}{0.998\text{g}} = 1.0020$

or $5\lambda = \ln 1.0020$ or $\lambda = \frac{\ln 1.0020}{5}\text{ y}^{-1}$

or $T_{1/2} = \frac{\ln 2}{\lambda} = \frac{5\ln 2}{\ln 1.002} = 1735$ y

28. (a) : Refer to figure.



The sphere is at equilibrium, if

$T\sin\theta = qE$

$T\cos\theta = mg$

$\therefore$ $\frac{T\sin\theta}{T\cos\theta} = \frac{qE}{mg}$

or, $E = \frac{mg\tan\theta}{q}$

$\therefore$ $V = Ed = \frac{mgd\tan\theta}{q}$

$= \frac{2\times10^{-4}\times9.8\times5\times10^{-2}\times\tan30°}{6\times10^{-9}}$

$= 9.4\times10^3$ V

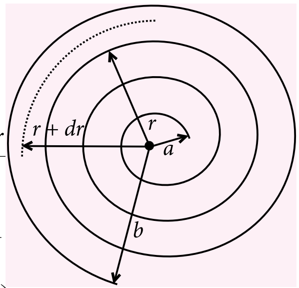
29. (c) : When the ball dropped, its acceleration is g as is observed by a man standing stationary on the ground. Now the man inside the lift has downward acceleration a, the acceleration of the ball as observed by this man will be $g - a$.

30. (c) : Consider an element of thickness (dr) at a distance r from the centre of spiral - coil.

$\therefore$ Number of turns in spiral = N

$\therefore$ Turns per unit length = $\frac{N}{(b-a)}$

Magnetic field due to element = dB



$\therefore$ $dB = \frac{\mu_0(dN)I}{2r}$

or $dB = \frac{\mu_0 I}{2}\frac{N}{(b-a)}\frac{dr}{r}$

or $\int dB = \frac{\mu_0 NI}{2(b-a)}\int_a^b \frac{dr}{r}$

or $B = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$.

31. (a) : Let x be each side of the cube.

As area (A) of each face is 4 cm^2,

$A = x\times x = 4$ cm^2 or $x = 2$ cm

Further, with usual notation,

$(T_1 - T_2) = (100°\text{C} - 0°\text{C}) = 100\text{C}°$

(T_1 = temperature of steam, T_2 = temperature of melting ice)

$k = 0.2 \text{ cal s}^{-1}\text{ cm}^{-1}\text{ C}^{o-1}$, t = 10 min = 10 × 60 s = 600 s

As $Q = \frac{kA(T_1 - T_2)t}{x}$,

$Q = \frac{0.2\times4\times100\times600}{2}$ cal = 24,000 cal

If m gram of ice be melted with this heat and L be the latent heat of fusion of ice,

$Q = mL$

or $m = \frac{Q}{L} = \frac{24,000}{80}$ (as $L = 80 \text{ cal g}^{-1}$)

or $m = 300$ g = 0.3 kg

32. (c) : Here $u_1 = u$, $u_2 = 0$

$$\therefore \quad e = \frac{v_2 - v_1}{u_2 - u_1} = \frac{v_2 - v_1}{u - 0}$$

or $\quad v_2 - v_1 = eu \quad$ ...(i)

By the law of conservation of momentum,

$$mu + m \times 0 = mv_1 + mv_2$$

or $v_1 + v_2 = u \quad$ ...(ii)

Adding (i) and (ii),

$$2v_2 = u + eu = u(1 + e)$$

or $$v_2 = \frac{u(1+e)}{2}$$

Again, from (ii),

$$v_1 = u - v_2 = u - \frac{u(1+e)}{2} = \frac{u(1-e)}{2}$$

Hence, $$\frac{v_2}{v_1} = \frac{1+e}{1-e}.$$

33. (a) : $I_{rms} = \dfrac{12\text{ W}}{24\text{ V}} = \dfrac{1}{2}\text{A} \qquad \left(\text{as } I = \dfrac{P}{V}\right)$

$$I_0 = I_{rms}\sqrt{2} = \left(\frac{1}{2}A\right)\sqrt{2} = \frac{1}{\sqrt{2}}\text{A}$$

34. (b): As $\upsilon_1 = \dfrac{1}{2\pi}\sqrt{\dfrac{k_1}{m}}$, $\upsilon_2 = \dfrac{1}{2\pi}\sqrt{\dfrac{k_2}{m}}$,

Frequency of the system,

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k_1}{m} + \frac{k_2}{m}} = \frac{1}{2\pi}\sqrt{(2\pi\upsilon_1)^2 + (2\pi\upsilon_2)^2}$$

$$= \sqrt{\upsilon_1^2 + \upsilon_2^2}$$

35. (d): Here, $r = 1.00$ mm $= 1.00 \times 10^{-3}$ m

$T = 0.465$ N m^{-1}

$\rho = 13.6 \times 10^3$ kg m^{-3}, $\theta = 140°$

If h is the difference in the level of mercury in the tube and the surface outside,

$$h = \frac{2T\cos\theta}{r\rho g}$$

or $$h = \frac{2(0.462\text{ N m}^{-1})(-0.7660)}{(1.00\times10^{-3}\text{m})(13.6\times10^3\text{kg m}^{-3})(9.8\text{ m s}^{-2})}$$

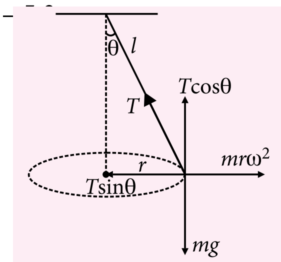
$= -5.3 \times 10^{-3}$ m = –

36. (a) : For equilibrium,

$T\sin\theta = mr\omega^2$

$$T \times \frac{r}{l} = mr\left(2\pi \cdot \frac{2}{\pi}\right)^2$$

$T = 16\, ml$



37. (b): $\upsilon = 9500$ Hz, $\upsilon_s = ?$, $v = 300$ m s^{-1}, $\upsilon' = 10000$ Hz

As the source is approaching a stationary observer,

$$\upsilon' = \upsilon\left(\frac{v'}{v - v_s}\right) \text{ or } 10000 = 9500\left(\frac{300}{300 - \upsilon_s}\right)$$

$v_s = 15$ m s^{-1}

38. (c) : Here; $C = 8.00\ \mu$F $= 8.00 \times 10^{-6}$ F, $\upsilon = 3.00$ kHz, $V_0 = 30.0$ V

Clearly, $\omega = 2\pi\upsilon = 2\pi \times (3.00 \times 10^3\text{ s}^{-1})$
$= 6\pi \times 10^3$ s^{-1}

Voltage across the capacitor, $V = V_0 \sin\omega t$
$= (30.0\text{ V})\sin(6\pi \times 10^3\, t)$

Displacement current, I_d

$$= \frac{dQ}{dt} = \frac{d}{dt}(Q) = \frac{d}{dt}(CV) = C\frac{dV}{dt}$$

$$= (8.00 \times 10^{-6})\frac{d}{dt}[30.0\sin(6\pi \times 10^3\, t)]$$

$= (8.00 \times 10^{-6})(30.0)(6\pi \times 10^3)\cos(6\pi \times 10^3\, t)$

$= (8.00 \times 10^{-6})(30.0)(6\pi \times 10^3)\cos(6\pi \times 10^3\, t)$

$= (4.52\text{ A})\cos(6\pi \times 10^3\, t)$

Hence, the displacement current varies sinusoidally with time and has a maximum value of 4.52 A.

39. (c) : P-V diagram represents, PV = constant (Boyle's law) This implies that temperature (T) is constant. Further, from 1 to 2, pressure decreases. Thus, corresponding T-P diagram is represented by (iii).

40. (a) : Here $m = 2$ kg, $R = 0.1$ m

Height of inclined plane, $h = 4$ m

At the top of the inclined plane, the cylinder has P.E. $= mgh$

At the bottom of the inclined plane, the cylinder has translational K.E. $\left(= \dfrac{1}{2}mv^2\right)$ and rotational K.E. $\left(= \dfrac{1}{2}I\omega^2\right)$

By conservation of energy,

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

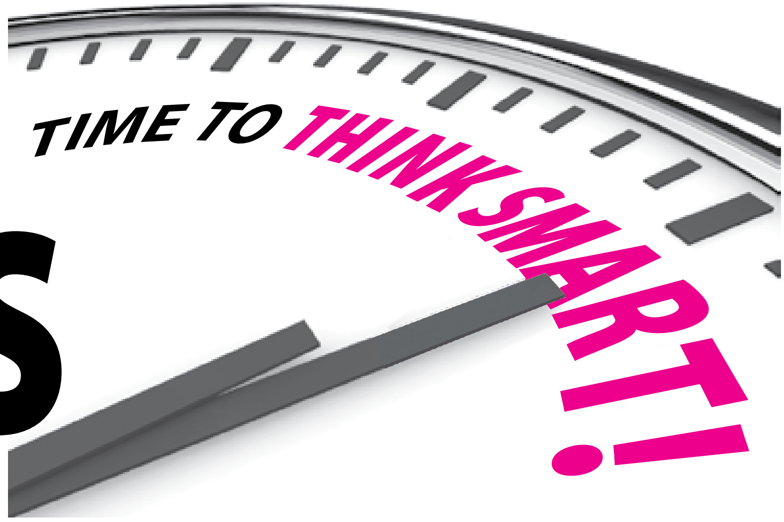
But $v = R\omega$ and $I = \dfrac{1}{2}mR^2$

$$\therefore \quad \frac{1}{2}m(R\omega)^2 + \frac{1}{2} \times \frac{1}{2}mR^2\omega^2 = mgh$$

or $$\frac{3}{4}mR^2\omega^2 = mgh \text{ or } \omega^2 = \frac{4gh}{3R^2}$$

$$\therefore \quad \text{Rotational K.E.} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{2}mR^2 \times \frac{4gh}{3R^2}$$

$$= \frac{mgh}{3} = \frac{2 \times 9.8 \times 4}{3} = 26.13\text{ J.}$$



PRACTICE PAPER AIIMS

Exam on : 1st June

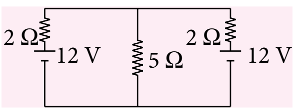
1. A car travels equal distances in the same direction with velocities 60 km h^{-1}, 20 km h^{-1} and 10 km h^{-1} respectively. The average velocity of the car over the whole journey of motion is
(a) 8 m s^{-1} (b) 7 m s^{-1}
(c) 6 m s^{-1} (d) 5 m s^{-1}

2. Moment of inertia of a thin rod of mass M and length L about an axis passing through its centre is $\frac{ML^2}{12}$. Its moment of inertia about a parallel axis at a distance of $\frac{L}{4}$ from this axis is given by
(a) $\frac{ML^2}{48}$ (b) $\frac{ML^3}{48}$ (c) $\frac{ML^2}{12}$ (d) $\frac{7ML^2}{48}$

3. Two linear SHMs of equal amplitude A and angular frequencies ω and 2ω are impressed on a particle along the axes x and y respectively. If the initial phase difference between them is $\pi/2$, the resultant path followed by the particle is
(a) $y^2 = x^2(1 - x^2/A^2)$ (b) $y^2 = 2x^2(1 - x^2/A^2)$
(c) $y^2 = 4x^2(1 - x^2/A^2)$ (d) $y^2 = 8x^2(1 - x^2/A^2)$

4. Equal charges, q each are placed at the vertices A and B of an equilateral triangle ABC of side a. The magnitude of electric intensity at the point C is
(a) $\frac{q}{4\pi\varepsilon_0 a^2}$ (b) $\frac{\sqrt{2}\,q}{4\pi\varepsilon_0 a^2}$
(c) $\frac{\sqrt{3}\,q}{4\pi\varepsilon_0 a^2}$ (d) $\frac{2q}{4\pi\varepsilon_0 a^2}$

5. In the arrangement shown in figure, the current through 5 Ω resistor is



(a) 2 A (b) 1 A (c) $\frac{12}{7}$ A (d) zero

6. The greatest length of steel wire that can hang vertically without breaking is
(Given, breaking stress of steel = 8.0×10^8 N m^{-2}, Density of steel = 8.0×10^3 kg m^{-3})
(a) 10 m (b) 10^2 m (c) 10^3 m (d) 10^4 m

7. A rectangular vessel when full of water, takes 10 min to be emptied through an orifice in its bottom. How much time will it take to be emptied when half filled with water?
(a) 9 min (b) 7 min (c) 5 min (d) 3 min

8. If the frequency of sound produced by a siren increases from 400 Hz to 1200 Hz while the wave amplitude remains constant, the ratio of the intensity of the 1200 Hz to that of the 400 Hz wave will be
(a) 1 : 1 (b) 1 : 3 (c) 3 : 1 (d) 9 : 1

9. A spherical charged conductor has surface charge density σ. The electric field on its surface is E and electric potential of conductor is V. Now the radius of the sphere is halved keeping the charge to be constant. The new values of electric field and potential would be
(a) $2E, 2V$ (b) $4E, 2V$
(c) $4E, 4V$ (d) $2E, 4V$

10. In a series LCR circuit, the voltage across the resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited, the voltage across the inductance will be
(a) 10 V (b) $10\sqrt{2}$ V
(c) $\frac{10}{\sqrt{2}}$ V (d) 20 V

11. Light of two different frequencies whose photons have energies 1 eV and 2.5 eV respectively, successively illuminate a metal whose work function is 0.5 eV. The ratio of the maximum speeds of the emitted electrons will be
(a) 1 : 5 (b) 1 : 4 (c) 1 : 2 (d) 1 : 1

12. If the series limit wavelength of the Lyman series for hydrogen atom is 912 Å, then the series limit wavelength for the Balmer series for the hydrogen atom is

(a) 912 Å (b) 912 × 2 Å

(c) 912 × 4 Å (d) $\frac{912}{2}$ Å

13. A freshly prepared radioactive source of half-life 2 hours emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with the source is

(a) 6 hours (b) 12 hours

(c) 24 hours (d) 128 hours

14. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if

(a) both the length and radius of the wire are halved

(b) both the length and radius of the wire are doubled

(c) the radius of the wire is doubled

(d) the length of the wire is doubled.

15. A bar magnet when placed at an angle of 30° to the direction of magnetic field induction of 5×10^{-2} T, experience a moment of couple 2.5×10^{-6} N m. If the length of the magnet is 5 cm, its pole strength is

(a) 2×10^{2} A m (b) 2×10^{-3} A m

(c) 5 A m (d) 5×10^{-2} A m

16. A concave lens forms the image of an object such that the distance between the object and image is 10 cm and the magnification produced is 1/4. The focal length of the lens will be

(a) 8.6 cm (b) 6.2 cm (c) 10 cm (d) 4.4 cm

17. In Young's double slit experiment, the spacing between the slits is d and wavelength of light used is 6000 Å. If the angular width of a fringe formed on a distant screen is 1°, then value of d is

(a) 1 mm (b) 0.05 mm

(c) 0.03 mm (d) 0.01 mm

18. An oil drop of mass 50 mg and of charge −5 μC is just balanced in air against the force of gravity. The strength of the electric field required to balance, is (Take $g = 9.8$ m s^{-2})

(a) 98 N C^{-1} upwards

(b) 98 N C^{-1} downwards

(c) 9.8 N C^{-1} towards north

(d) 9.8 N C^{-1} towards south

19. A potential difference of 2 V is applied between the opposite faces of a Ge crystal plate of area 1 cm^2 and thickness 0.5 mm. If the concentration of electrons in Ge is 2×10^{19} m^{-3} and mobilities of electrons and holes are 0.36 m^2 V^{-1} s^{-1} and 0.14 m^2 V^{-1} s^{-1} respectively, then the current flowing through the plate will be

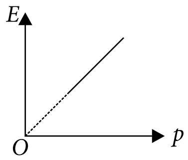
(a) 0.25 A (b) 0.45 A (c) 0.56 A (d) 0.64 A

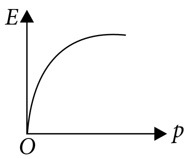
20. At 1 atm pressure, 1 g of water having a volume of 1000 cm^3 becomes 1671 cm^3 of steam when boiled. The heat of vaporization of water at 1 atm is 539 cal g^{-1}. What is the change in internal energy during the process?

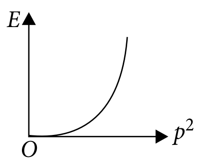
(a) 539 cal (b) 417 cal

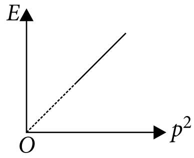
(c) 498.5 cal (d) 835.5 cal

21. A particle is projected from a point at a certain angle with the horizontal. At any instant t, if p is the linear momentum and E the kinetic energy, then which of the following graphs is correct?

(a) 

(b) 

(c) 

(d) 

22. A man measures the period of a simple pendulum inside a stationary lift and finds it to be T second. If the lift accelerates upwards with an acceleration $\frac{g}{4}$, then the period of the pendulum will be

(a) T (b) $\frac{T}{4}$ (c) $\frac{2T}{\sqrt{5}}$ (d) $2\sqrt{5}T$

23. A pendulum bob of mass 10^{-2} kg is raised to a height 5×10^{-2} m and then released. At the bottom of its swing, it picks up a mass 10^{-3} kg. To what height will the combined mass rise?

(a) 4.1×10^{-2} m (b) 2.5×10^{-2} m

(c) 1.3×10^{-2} m (d) 0.7×10^{-2} m

24. A rod of length 1.0 m and mass 0.5 kg fixed at one end is initially hanging vertical. The other end is now raised until it makes an angle 60° with the vertical. The work required to be done for this, is

(a) 1.2 J (b) 1.8 J

(c) 2.4 J (d) 3.0 J

25. A common emitter transistor amplifier has a current gain of 50. If the load resistance is 4 kΩ and input resistance is 500 Ω, the voltage gain of the amplifier is

(a) 200 (b) 400 (c) 600 (d) 800

26. When an electron jumps from the fourth orbit to the second orbit, one gets the

(a) second line of Paschen series
(b) second line of Balmer series
(c) first line of Pfund series
(d) second line of Lyman series

27. The critical angle of a certain medium is $\sin^{-1}\left(\frac{3}{5}\right)$. The polarising angle of the medium is

(a) $\sin^{-1}\left(\frac{4}{5}\right)$ (b) $\tan^{-1}\left(\frac{5}{3}\right)$

(c) $\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

28. Which of the following types of electromagnetic radiation travels at the greatest speed in vacuum?

(a) Radio waves (b) Visible light
(c) X-rays
(d) All of these travel at the same speed

29. A vector $\vec{A}$ is along the positive z-axis and its vector product with another vector $\vec{B}$ is zero, then vector $\vec{B}$ could be

(a) $\hat{i}+\hat{j}$ (b) $4\hat{i}$ (c) $\hat{j}+\hat{k}$ (d) $-7\hat{k}$

30. Two perfect gases having masses m_1 and m_2 at temperatures T_1 and T_2 respectively are mixed without any loss of energy. If the molecular weights of the gases are M_1 and M_2 respectively, then the final temperature of the mixture is

(a) $\frac{(m_1T_1+m_2T_2)}{(m_1+m_2)}$ (b) $\frac{(M_1T_1+M_2T_2)}{(M_1+M_2)}$

(c) $\frac{\left(\frac{m_1T_1}{M_1}\right)+\left(\frac{m_2T_2}{M_2}\right)}{\left(\frac{m_1}{M_1}\right)+\left(\frac{m_2}{M_2}\right)}$ (d) $\frac{\left[\frac{M_1T_1}{m_1}+\frac{M_2T_2}{m_2}\right]}{\left[\left(\frac{M_1}{m_1}\right)+\left(\frac{M_2}{m_2}\right)\right]}$

31. A proton with energy of 2 MeV enters a region of a uniform magnetic field of 2.5 T normally. The magnetic force on the proton is
(Take, mass of proton to be 1.6×10^{-27} kg)

(a) 3×10^{-12} N (b) 3×10^{-8} N
(c) 8×10^{-12} N (d) 2×10^{-10} N

32. The power and type of lens by which a person can see clearly the distant objects, if the person cannot see objects beyond 40 cm, are

(a) –2.5 D and concave lens
(b) –2.5 D and convex lens
(c) –3.5 D and concave lens
(d) –3.5 D and convex lens

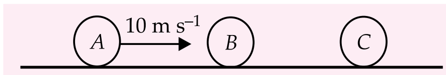
33. A reactor is developing energy at the rate of 3000 kW. How many atoms of U^{235} undergo fission per second, if 200 MeV energy is released per fission?

(a) 6.5×10^{22} (b) 5.15×10^{21}
(c) 3.384×10^{23} (d) 9.4×10^{16}

34. Power supplied to a particle of mass 2 kg varies with time as $P = \frac{3t^2}{2}$ W, where t is in seconds. If velocity of particle at $t = 0$ is zero, then the velocity of particle at $t = 2$ s will be

(a) 1 m s^{-1} (b) 4 m s^{-1}
(c) 2 m s^{-1} (d) $2\sqrt{2}$ m s^{-1}

35. Three identical spherical balls A, B and C are placed on a table as shown in the figure along a straight line.



B and C are at rest initially. The ball A hits B head on with a speed of 10 m s^{-1}. Then after all collisions (assumed to be elastic) A and B come to rest and C takes off with a velocity of

(a) 5 m s^{-1} (b) 10 m s^{-1}
(c) 2.5 m s^{-1} (d) 7.5 m s^{-1}

36. The time period of an artificial satellite in a circular orbit of radius R is 2 days and its orbital velocity is v_o. If time period of another satellite in a circular orbit is 16 days then

(a) its radius of orbit is $4R$ and orbital velocity is v_o.
(b) its radius of orbit is $4R$ and orbital velocity is $\frac{v_o}{2}$.
(c) its radius of orbit is $2R$ and orbital velocity is v_o.
(d) its radius of orbit is $2R$ and orbital velocity is $\frac{v_o}{2}$.

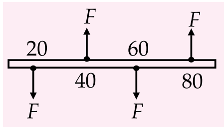
37. Force constant of a spring (k) is analogous to

(a) $\frac{YA}{L}$ (b) $\frac{YL}{A}$ (c) $\frac{AL}{Y}$ (d) ALY

38. Two rings of same radius r and mass m are placed such that their centres are at a common point and their planes are perpendicular to each other. The moment of inertia of the system about an axis passing through the centre and perpendicular to plane of one of the rings is

(a) $\frac{1}{2}mr^2$ (b) mr^2 (c) $\frac{3}{2}mr^2$ (d) $2mr^2$

39. Four equal and parallel forces are acting on a rod of length 100 cm, as shown in figure, at distances of 20 cm, 40 cm, 60 cm and 80 cm respectively from one end of the rod. Under the influence of these forces, the rod (neglecting its weight)



(a) experiences no torque
(b) experiences torque
(c) experiences a linear motion
(d) experiences torque and also a linear motion

40. The current gain of a transistor in a common base arrangement is 0.98. Find the change in collector current corresponding to a change of 5.0 mA in emitter current. What would be the change in base current?

(a) 4.9 mA, 0.1 mA (b) 4.9 mA, 0.2 mA
(c) 5.9 mA , 0.3 mA (d) 5.9 mA, 0.8 mA

Directions : In the following questions (41-60), a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

41. Assertion : During rapid pumping of air in tyres, air inside the tyre is hotter than atmosphere air.
Reason : Adiabatic process occurs at very high rate.

42. Assertion: When a cyclist doubles his speed of turning, the chance of his skidding becomes nearly four times.
Reason : When the speed of the vehicle increases, the angle of bending also increases.

43. Assertion: At higher temperatures, the peak emission wavelength of a black body shifts to lower wavelengths.
Reason: The peak emission wavelength of a black body is proportional to the fourth power of absolute temperature.

44. Assertion : Work done by a gas in isothermal expansion is more than the work done by the gas in the same expansion, adiabatically.
Reason : Temperature remains constant in isothermal expansion, and not in adiabatic expansion.

45. Assertion : A current carrying conductor produces only an electric field.
Reason : Electrons in motion give rise to only an electric field.

46. Assertion : The energy stored in the inductor of 2 H, when a current of 10 A flows through it is 100 J.
Reason : Energy stored in an inductor is directly proportional to its inductance.

47. Assertion : The electric field and hence electric field lines are everywhere at right angle to an equipotential surface.
Reason : Equipotential surfaces are closer together where the electric field is stronger and farther apart where the field is weaker.

48. Assertion : When temperature difference across the two sides of a wall is increased, its thermal conductivity remains constant.
Reason : Thermal conductivity depends on nature of material of the wall.

49. Assertion : Young's modulus for a perfectly plastic body is zero.
Reason : For a perfectly plastic body, restoring force is zero.

50. Assertion : Orbit of a satellite is within the gravitational field of earth whereas escaping is beyond the gravitational field of earth.
Reason : Orbital velocity of a satellite is greater than its escape velocity.

51. Assertion: A piece of ice, with a stone frozen inside it, floats on water in a beaker. When the ice melts, the level of water in the beaker decreases.
Reason : Density of stone is more than that of water.

52. Assertion : Adding a scalar to a vector of the same dimensions is a meaningful algebraic operation.
Reason : The displacement can be added with distance.

53. Assertion: A *p-n* junction with reverse bias can be used as a photodiode to measure light intensity.
Reason : In a reverse bias condition the current though small is more sensitive to change with incident light intensity.

54. Assertion : Most amplifiers use common emitter circuit configuration.
Reason : Its input resistance is comparatively higher.

55. Assertion : Magnetic force between two short magnets, when they are co-axial follows inverse square law of distance.
Reason : The magnetic forces between two poles do not follow inverse square law of distance.

56. Assertion: A photon has no rest mass, yet it carries definite momentum.
Reason: Momentum of photon is due to its energy and hence its equivalent mass.

57. Assertion : A capacitor of suitable capacitance can be used in an A.C. circuit in place of the choke coil.
Reason : A capacitor blocks D.C. and allows A.C. only.

58. Assertion : Static crashes are heard on radio, when a lightning flash occurs.
Reason : Light and radio waves travel with the same speed.

59. Assertion : Diffraction is common in sound but not common in light waves.
Reason : Wavelength of light is more than the wavelength of sound.

60. Assertion : Improper banking of roads causes wear and tear of tyres.
Reason : The necessary centripetal force is provided by the force of friction between the tyres and the road.

SOLUTIONS

1. (d) : Let the total distance be $3x$ km.

Then, average velocity $= \dfrac{\text{total distance}}{\text{total time taken}}$

$$= \frac{3x}{\frac{x}{60}+\frac{x}{20}+\frac{x}{10}} = \frac{3x}{\frac{x+3x+6x}{60}}$$

$$= \frac{3x\times 60}{10x} = 18\text{ km h}^{-1} = \frac{18\times 5}{18}\text{ m s}^{-1} = 5\text{ m s}^{-1}$$

2. (d) : Applying parallel axis theorem
$I = I_{CM} + Md^2$

$$= \frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 = \frac{7ML^2}{48}$$

3. (c) : $x = A\sin(\omega t + \pi/2) = A\cos\omega t$

$\therefore$ $\cos\omega t = \dfrac{x}{A}$ and $\sin\omega t = \sqrt{1-\left(\dfrac{x^2}{A^2}\right)}$

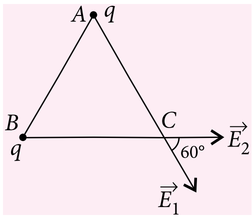
$y = A\sin 2\omega t = 2A\sin\omega t\cos\omega t$
or $y^2 = 4A^2\sin^2\omega t\cos^2\omega t$

$$= 4A^2\times\frac{x^2}{A^2}\times\left(\frac{A^2-x^2}{A^2}\right) = 4x^2\left(1-\frac{x^2}{A^2}\right)$$

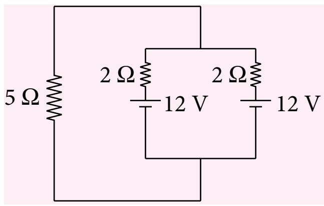
4. (c) : $E_1 = E_2 = \dfrac{q}{4\pi\varepsilon_0 a^2}$, acting at 60°

$\therefore$ Resultant intensity

$$E = \sqrt{E_1^2+E_2^2+2E_1E_2\cos\theta}$$
$$= \sqrt{E_1^2+E_1^2+2E_1^2\cos 60^\circ}$$
$$= E_1\sqrt{3} = \frac{q\sqrt{3}}{4\pi\varepsilon_0 a^2}$$



5. (a) : The circuit may be redrawn as shown in the figure.



Here, $\varepsilon_{eq} = 12\text{ V}, r_{eq} = \dfrac{2\times 2}{2+2} = 1\ \Omega$

$$\therefore\quad I = \frac{\varepsilon_{eq}}{R+r_{eq}} = \frac{12}{5+1} = \frac{12}{6} = 2\text{ A}$$

6. (d) : Let l be the length of the wire that can hang vertically without breaking. Then the stretching force on it is equal to its own weight. If A is the area of cross-section and ρ the density, then

maximum stress $(\sigma_m) = \dfrac{\text{weight}}{A}$ or $\sigma_m = \dfrac{(Al\rho)g}{A}$

$$\therefore\quad l = \frac{\sigma_m}{\rho g}$$

Substituting the values, we get

$$l = \frac{8.0\times 10^8}{(8.0\times 10^3)(10)} = 10^4\text{ m}$$

7. (b) : If A_0 is the area of orifice at the bottom below the free surface and A that of vessel, time t taken to be emptied the tank,

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}} \quad \therefore \quad \frac{t_1}{t_2} = \sqrt{\frac{H_1}{H_2}} = \sqrt{\frac{H_1}{H_1/2}}$$

$$\Rightarrow \frac{t_1}{t_2} = \sqrt{2} \quad \therefore \quad t_2 = \frac{t_1}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \approx 7 \text{ min}$$

8. (d) : $I = \frac{1}{2}\rho\omega^2 A^2 v$ or $I \propto \omega^2$ or $I \propto \upsilon^2$

$$\therefore \quad \frac{I_2}{I_1} = \left(\frac{\upsilon_2}{\upsilon_1}\right)^2 = \left(\frac{1200}{400}\right)^2 = 9$$

9. (b) : $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

As q is constant, so $E \propto \frac{1}{R^2}$

Radius is halved. Therefore electric field will become 4 times or 4E.

Further, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

As q is constant, so $V \propto \frac{1}{R}$

Radius is halved, so potential will become two times or 2V.

10. (c) : Here, $R = X_L = X_C$

(∵ voltage across them is same)

Total voltage in the circuit,

$V = I[R^2 + (X_L - X_C)^2]^{1/2}$

$= IR = 10$ V

When capacitor is short circuited,

$$I' = \frac{V}{(R^2 + X_L^2)^{1/2}} = \frac{10}{\sqrt{2}R}$$

∴ Potential drop across inductance

$$= I'X_L = I'R = \frac{10}{\sqrt{2}} \text{ V}$$

11. (c) : If E is the energy of incident photon and ϕ_0 the work function, then according to Einstein's photoelectric equation,

$$E - \phi_0 = \frac{1}{2}mv^2 \text{ or } v = \sqrt{\frac{2(E-\phi_0)}{m}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{E_1 - \phi_0}{E_2 - \phi_0}} = \sqrt{\frac{1-0.5}{2.5-0.5}} = \sqrt{\frac{0.5}{2}} = \frac{1}{2}$$

12. (c) : For series limit of Balmer series,

$n_i = \infty$, $n_f = 2$

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] \text{ or } \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{R}{4}$$

$$\therefore \quad \lambda = \frac{4}{R} = 4 \times 912 \text{ Å}$$

(∵ series limit of Lyman series = $1/R$ = 912 Å)

13. (b) : Let safe level activity be A.

Initial activity = $64A$.

Hence, $\frac{N}{N_0} = \frac{A}{A_0} = \frac{A}{64A} = \frac{1}{64}$

or $\left(\frac{1}{2}\right)^n = \frac{1}{64} = \left(\frac{1}{2}\right)^6$ or $n = 6$

$\Rightarrow \frac{t}{T} = 6 \quad \therefore t = 12$ hours (∵ T = 2 hours)

14. (b) : Heat developed, $H = \frac{V^2}{R} \times t$

Heat developed will be doubled when R is halved.

Further, $R = (\rho l/\pi r^2)$

$$\therefore \quad H = \frac{V^2 \times \pi r^2 \times t}{\rho l}$$

So, heat produced will be doubled when both the length and radius of the wire are doubled.

15. (b) : Here, $\theta = 30°$, $B = 5 \times 10^{-2}$ T,

$\tau = 2.5 \times 10^{-6}$ N m, $2l = 5$ cm = 0.05 m

$\tau = MB \sin\theta = m(2l)B \sin\theta$

$$\therefore \quad m = \frac{\tau}{B(2l)\sin\theta} = \frac{2.5 \times 10^{-6}}{5 \times 10^{-2}(0.05)\sin 30°}$$

$m = 2 \times 10^{-3}$ A m

16. (d) : Concave lens forms the virtual image of a real object.

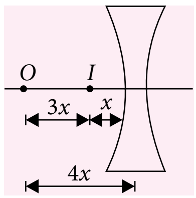
Let $u = -4x$, then $v = -x$ $\left(\because m = \frac{1}{4}\right)$

As per question, $3x = 10$ cm

or $x = \frac{10}{3}$ cm

$\therefore \quad u = -\frac{40}{3}$ cm and

$v = -\frac{10}{3}$ cm



Substituting in lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \frac{1}{f} = \frac{-3}{10} + \frac{3}{40} \text{ or } f = -\frac{40}{9} = -4.4 \text{ cm}$$

17. (c) : Here, $\sin\theta \simeq \theta = \left(\frac{y}{D}\right)$

So, $\Delta\theta = \frac{\Delta y}{D}$

For $\Delta y = \beta$, angular fringe width $\theta_0 = \Delta\theta$

$$\therefore \quad \theta_0 = \frac{\beta}{D} = \frac{D\lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d}$$

As $\theta_0 = 1° = \frac{\pi}{180}$ rad and $\lambda = 6 \times 10^{-7}$ m

$$\therefore \quad d = \frac{\lambda}{\theta_0} = \frac{180}{\pi} \times 6 \times 10^{-7}$$

$= 3.44 \times 10^{-5}$ m ≈ 0.03 mm

18. (b) : As $qE = mg$

$$\text{or} \quad E = \frac{mg}{q} = \frac{(50 \times 10^{-6}\,\text{kg}) \times 9.8\,\text{m s}^{-2}}{5 \times 10^{-6}\,\text{C}} = 98\,\text{N C}^{-1}$$

Since the force due to electric field on charged particle should be opposite to the gravity pull and charge on the drop is negative, hence the electric field must act vertically downwards.

19. (d) : Conductivity, $\sigma = ne(\mu_e + \mu_h)$

$= 2 \times 10^{19} \times 1.6 \times 10^{-19} \times (0.36 + 0.14)$

$= 1.6\ \Omega^{-1}\ \text{m}^{-1}$

$$\therefore \quad \text{Resistance, } R = \rho\frac{l}{A} = \frac{l}{\sigma A} = \frac{0.5 \times 10^{-3}}{1.6 \times 10^{-4}} = \frac{25}{8}\,\Omega$$

$$\text{Hence, current } I = \frac{V}{R} = \frac{2}{25/8} = \frac{16}{25}\,\text{A} = 0.64\,\text{A}$$

20. (c) : Heat spent during vaporisation

$Q = mL = 1 \times 539 = 539$ cal

Work done $W = P(V_v - V_l)$

$= 1.013 \times 10^5 \times (1671 - 1) \times 10^{-6}$ J

$= \frac{169.2}{4.18}$ cal $= 40.5$ cal

$\therefore$ Change in internal energy

$U = 539$ cal $- 40.5$ cal $= 498.5$ cal

21. (d) : As $p^2 = 2Em$ or $p^2 \propto E$

i.e., p^2 versus E graph is a straight line passing through origin.

22. (c) : In stationary lift, $T = 2\pi\sqrt{\frac{l}{g}}$

In upward moving lift,

$$T' = 2\pi\sqrt{\frac{l}{(g+a)}} \quad \text{where } a = \text{acceleration of lift}$$

$$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{g}{\left(g + \frac{g}{4}\right)}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\therefore \quad T' = \frac{2T}{\sqrt{5}}$$

23. (a) : Velocity of pendulum bob in mean position

$$v_1 = \sqrt{2gh_1} = \sqrt{2 \times 10 \times 5 \times 10^{-2}} = 1\,\text{m s}^{-1}$$

When the bob picks up a mass 10^{-3} kg at the bottom, then by conservation of linear momentum the velocity of combined mass is given by

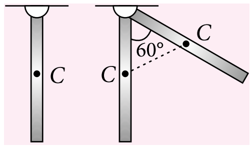
$m_1v_1 + m_2v_2 = (m_1 + m_2)v$

$10^{-2} \times 1 + 10^{-3} \times 0 = (10^{-2} + 10^{-3})v$

$$\text{or} \quad v = \frac{10^{-2}}{1.1 \times 10^{-2}} = \frac{10}{11}\,\text{m s}^{-1}$$

$$\text{Now, } h = \frac{v^2}{2g} = \frac{(10/11)^2}{2 \times 10} = 4.1 \times 10^{-2}\,\text{m}$$

24. (a) :



W = Change in potential energy

$$= mg\frac{l}{2}(1 - \cos\theta)$$

Substituting the values, we have

$$W = (0.5)(9.8)\left(\frac{1.0}{2}\right)(1 - \cos 60°) = 1.2\,\text{J}$$

25. (b) : $R_L = 4$ kΩ, $R_I = 500\ \Omega$, $\beta = \frac{I_C}{I_B} = 50$

$V_{CE} = I_CR_L$, $V_{BE} = I_BR_I$

$$A_V = \frac{V_{CE}}{V_{BE}} = \frac{I_C}{I_B} \times \frac{R_L}{R_I} = 50 \times \frac{4 \times 10^3}{500} = 400$$

26. (b) : Electron jump, to second orbit leads to Balmer series. When an electron jumps from 4th orbit to 2nd orbit, it gives rise to second line of Balmer series.

27. (b) : Given, $\sin C = \frac{3}{5}$. $\therefore\ \mu = \frac{1}{\sin C} = \frac{5}{3}$

If i_p is the polarising angle, then according to Brewster's law

$$\tan i_p = \mu \quad \text{or} \quad i_p = \tan^{-1}(\mu) = \tan^{-1}\left(\frac{5}{3}\right)$$

28. (d) : In vacuum, all electromagnetic waves travel with the same speed $c (= 3 \times 10^8\ \text{m s}^{-1})$.

29. (d) : The vector product of two non-zero vectors is zero if they are in the same direction or in the opposite direction. Hence vector $\vec{B}$ must be parallel or antiparallel to vector $\vec{A}$, *i.e.* along $\pm z$-axis.

30. (c) : Number of moles of gas 1, $n_1 = \frac{m_1}{M_1}$

Number of moles of gas 2, $n_2 = \frac{m_2}{M_2}$

As there is no loss of energy, the final temperature of the mixture is

$$T_{\text{mixture}} = \frac{n_1T_1 + n_2T_2}{n_1 + n_2} = \frac{\left(\frac{m_1}{M_1}T_1 + \frac{m_2}{M_2}T_2\right)}{\left(\frac{m_1}{M_1} + \frac{m_2}{M_2}\right)}$$

31. (c) : Here, $E = 2$ MeV $= 2 \times 1.6 \times 10^{-13}$ J

$B = 2.5$ T, $m = 1.6 \times 10^{-27}$ kg,

$\theta = 90°$, $q = 1.6 \times 10^{-19}$ C

As, $E = \frac{1}{2}mv^2$ or $v = \sqrt{\frac{2E}{m}}$

Now $F = qvB\sin\theta$

$$= q \times \sqrt{\frac{2E}{m}} \times B \times \sin 90° = q \times \sqrt{\frac{2E}{m}} \times B$$

$$= (1.6 \times 10^{-19}) \times \sqrt{\frac{2 \times 2 \times 1.6 \times 10^{-13}}{1.6 \times 10^{-27}}} \times 2.5$$

$$= 8 \times 10^{-12} \text{ N}$$

32. (a) : Here, in this case lens used by person should form the image of distant object at a distance of 40 cm in front of it.

$\therefore$ $u = -\infty$, $v = -40$ cm

and $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ or $\frac{1}{f} = \frac{1}{-40} - \frac{1}{-\infty}$

or $\frac{1}{f} = \frac{1}{-40}$ or $f = -40$ cm

$$\text{Power} = \frac{100}{f\,(\text{cm})} = \frac{100}{-40} = -2.5 \text{ D}$$

Negative sign shows that lens used is concave lens.

33. (d) : Rate of development of energy by the reactor

$= 3000$ kW $= 3 \times 10^6$ J s^{-1}

Energy released per fission = 200 MeV

$= 200 \times 1.6 \times 10^{-13}$ J $= 32 \times 10^{-12}$ J

Number of atoms undergoing fission per second

$$= \frac{3 \times 10^6}{32 \times 10^{-12}} = 9.4 \times 10^{16}$$

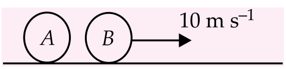
34. (c) : From work energy theorem, Δ K.E. $= W_{net}$

$\therefore$ $K_f - K_i = \int P\,dt$

$$\frac{1}{2}mv^2 - 0 = \int_0^2 \left(\frac{3}{2}t^2\right) dt \text{ , } \frac{1}{2}(2)v^2 = \frac{3}{2}\left[\frac{t^3}{3}\right]_0^2 = 4$$

$\therefore$ $v = 2$ m s^{-1}

35. (b) : Since A, B and C are identical balls, if any of the two balls undergo elastic collision, they will exchange their velocities.



Thus, when A and B collide, A comes to rest, and B starts moving ahead with 10 m s^{-1}.

Similarly, when B collides with C, B comes to rest and C starts moving ahead with a speed of 10 m s^{-1}.

36. (b) : According to Kepler's third law

$T^2 \propto R^3$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2}$$

$$\text{or} \quad R_2 = (R_1)\left(\frac{T_2}{T_1}\right)^{2/3} = (R_1)\left(\frac{16}{2}\right)^{2/3}$$

$= 4R_1 = 4R$ ...(i)

Orbital velocity, $v_o = \sqrt{\frac{GM}{R}}$

$$\therefore \quad \frac{v_{o2}}{v_{o1}} = \sqrt{\frac{R_1}{R_2}} = \sqrt{\frac{R}{4R}} = \frac{1}{2} \text{ or } v_{o2} = \frac{1}{2}v_{o1} = \frac{1}{2}v_o$$

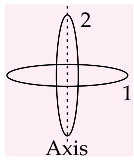
37. (a) : $Y = \frac{FL}{A\Delta L}$ or $F = \left(\frac{YA}{L}\right)\Delta L$

Comparing this with $F = k\Delta x$, we get, $k = \frac{YA}{L}$

38. (c) : $I_1 = mr^2$

I_2 = moment of inertia about the diameter

$= \frac{1}{2}mr^2$



Therefore, the moment of inertia of the system about the desired axis is $I = I_1 + I_2 = \frac{3}{2}mr^2$

39. (b) : Since, upward forces on the rod are equal to downward forces, hence, the resultant force on the rod is zero and there is no linear motion. Moment of forces about one end of rod

$= -F \times 20 + F \times 40 - F \times 60 + F \times 80 = F \times 40$

As it is not zero, hence a torque acts on the rod.

40. (a) : Given, $\alpha = 0.98$ and $\Delta I_E = 5.0$ mA

From the definition of, $\alpha = \frac{\Delta I_C}{\Delta I_E}$

Change in collector current

$\Delta I_C = (\alpha)(\Delta I_E) = 0.98 \times 5.0$ mA $= 4.9$ mA

Change in base current

$\Delta I_B = \Delta I_E - \Delta I_C = (5.0 - 4.9)$ mA $= 0.1$ mA

41. (a)

42. (b) : For the banking of curve, $\tan\theta = \frac{v^2}{rg}$. When velocity v is doubled, $\tan\theta$ becomes four times. With increase in θ, the chance of overturning also increases. When v increases, θ increases and consequently chance of skidding or overturning becomes greater.

43. (c) : According to Wien's displacement law, the maxima of wavelength emitted is inversely proportional to the absolute temperature $\left(\lambda_m \propto \frac{1}{T}\right)$.

44. (b) : Adiabatic curve is steeper than isothermal curve. Therefore, area under adiabatic curve is smaller than the area under isothermal curve *i.e.*, work done by the gas in adiabatic expansion is smaller than the work done by the gas in isothermal expansion. Assertion is true. Reason is also true but reason does not explain assertion.

45. (d) : A current carrying conductor has free electrons in motion. They give rise to a magnetic field. Electrons in motion in space produces both electric and magnetic fields. Thus, both assertion and reason are wrong.

46. (b) : The energy stored in the inductor is given by

$$U = \frac{1}{2}LI^2 = \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ J}$$

Energy stored is directly proportional to its inductance.

47. (b) : Electric field and field lines are perpendicular to an equipotential surface because there is no potential gradient along the equipotential surface *i.e.*, along the surface $\frac{dV}{dr} = 0$.

48. (a) : Thermal conductivity of the wall depends only on nature of material of the wall, and not on temperature difference across its two sides.

49. (a) : Young's modulus of a material, $Y = \frac{\text{stress}}{\text{strain}}$

and, $\text{stress} = \frac{\text{restoring force}}{\text{area}}$

As restoring force is zero for a perfectly plastic body,

$\therefore \quad Y = 0$

50. (c) : The orbital velocity, if a satellite close to earth is $v_o = \sqrt{gR_e}$, while the escape velocity for a body thrown from the earth's surface is $v_e = \sqrt{2gR_e}$

Thus $\frac{v_o}{v_e} = \sqrt{\frac{gR_e}{2gR_e}} = \frac{1}{\sqrt{2}}$ or $v_e = \sqrt{2}v_o$

51. (a) : If m_1 and m_2 are the respective masses of ice and stone, the volume of water displaced is $V = (m_1 + m_2)/\rho$. When ice melts, additional volume of water produced is m_1/ρ. The stone sinks in water and displaces a volume of water equal to its own volume, m_2/ρ_s where ρ_s is the density of stone.

Hence, total volume of extra water is $V' = \frac{m_1}{\rho} + \frac{m_2}{\rho_s}$.

As $\rho_s > \rho$, $\frac{1}{\rho_s} < \frac{1}{\rho}$.

Therefore, $V' < V$ and thus level of water in the beaker decreases.

52. (d) : Scalar and vector are quantities of different nature. We can add only those quantities which have same nature and units.

53. (a) : The reason that the reverse bias current is more sensitive to change is true. The current is due to minority carriers and is therefore small. The reason is the explanation of the assertion.

54. (b) : Most amplifiers use the common emitter circuit configuration because the circuit offers both current and voltage gains resulting in much higher power gain that can be obtained by a common-base amplifier. Input resistance of *CE* amplifier is higher than that of *CB* amplifier.

55. (d)

56. (a) : Equivalent mass of photon (m) is given from equation

$$E = mc^2 = h\upsilon \quad \therefore \quad m = \frac{h\upsilon}{c^2}$$

where E is energy, m is mass, c is speed of light, h is Planck's constant, υ is frequency.

$\therefore$ Momentum of photon $= \frac{h\upsilon}{c^2} \times c = \frac{h\upsilon}{c}$

57. (b) : We can use a capacitor of suitable capacitance as a choke coil, because average power consumed in an ideal capacitor is zero. Therefore, like a choke coil, a condenser can reduce A.C. without power dissipation.

58. (a)

59. (c) : For diffraction of a wave, size of an obstacle or aperture should be comparable to the size of wavelength of the wave. As wavelength of light is of the order of 10^{-6} m and obstacle / aperture of this size are rare, therefore, diffraction is not common in light waves. On the contrary, wavelength of sound is of the order of 1 m and obstacle / aperture of this size are readily available, therefore, diffraction is common in sound.

60. (a) : When roads are not properly banked, force of friction between tyres and road provides partially the necessary centripetal force. This cause wear and tear of tyres.

JIPMER PRACTICE PAPER

Exam on 7^{th} June

1. A body of mass 0.5 kg travels in a straight line with velocity $v = kx^{3/2}$ where $k = 5\ m^{-1/2}\ s^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is
(a) 1.5 J (b) 50 J (c) 10 J (d) 100 J

2. A ballet dancer spins with 2.8 rev s^{-1} with her arms out stretched. When the moment of inertia about the same axis becomes 0.7 times the earlier moment of inertia, the new rate of spin is
(a) 3.2 rev s^{-1} (b) 4.0 rev s^{-1}
(c) 4.8 rev s^{-1} (d) 5.6 rev s^{-1}

3. A bag of sand of mass m is suspended by a rope. A bullet of mass $m/20$ is fired at it with a velocity v and gets embedded into it. The velocity of the bag finally is
(a) $\frac{v}{21}$ (b) $\frac{v}{20}$ (c) $\frac{20v}{21}$ (d) $\frac{21v}{20}$

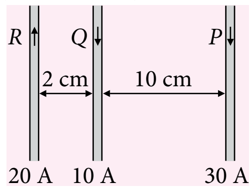
4. A car is moving on a road and rain is falling vertically. Select the correct answer.
(a) The rain will strike the hind screen only.
(b) The rain will strike the front screen only.
(c) The rain will strike both the screens.
(d) The rain will not strike any of the screens.

5. Two equal masses m and m are hung from a balance whose scale pan differs in vertical height by $h/2$. The error in weighing in terms of density of the earth ρ is
(a) $\frac{1}{3}\pi G\rho mh$ (b) $\pi G\rho mh$
(c) $\frac{4}{3}\pi G\rho mh$ (d) $\frac{8}{3}\pi G\rho mh$

6. Electric charges of 1 μC, –1 μC and 2 μC are placed in air at the corners A, B and C respectively of an equilateral triangle ABC having length of each side 10 cm. The resultant force on the charge at C is
(a) 0.9 N (b) 1.8 N (c) 2.7 N (d) 3.6 N

7. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be
(a) 300% (b) 200% (c) 100% (d) 50%

8. Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 cm length of wire Q is



(a) 1.4×10^{-4} N towards the right
(b) 1.4×10^{-4} N towards the left
(c) 2.6×10^{-4} N towards the right
(d) 2.6×10^{-4} N towards the left

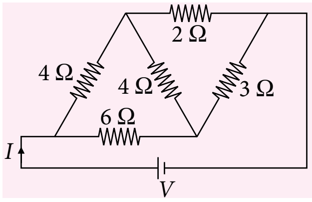
9. A magnetic field given by $B(t) = 0.2t - 0.05t^2$ tesla (where t denotes the time), is directed perpendicular to the plane of a circular coil containing 25 turns of radius 1.8 cm and whose total resistance is 1.5 Ω. The power dissipation at $t = 3$ s is approximately
(a) 1.37 μW (b) 7 μW
(c) zero (d) 4 μW

10. A prism, having refractive index $\sqrt{2}$ and refracting angle 30° has one of the refracting surfaces polished. A beam of light incident on the other refracting surface will retrace its path, if the angle of incidence is
(a) 0° (b) 30° (c) 45° (d) 60°

11. The wavelength of K_α line for an element of atomic number 43 is λ. Then the wavelength of K_α line for an element of atomic number 29 is
(a) $\frac{49}{29}\lambda$ (b) $\frac{42}{28}\lambda$ (c) $\frac{9}{4}\lambda$ (d) $\frac{4}{9}\lambda$

12. The reverse biasing in a p-n junction diode
(a) decreases the potential barrier
(b) increases the potential barrier
(c) increases the number of minority charge carriers
(d) increases the number of majority charge carriers

13. For a network shown in the figure, the value of the current I is



(a) $\frac{9V}{35}$ (b) $\frac{5V}{18}$ (c) $\frac{5V}{9}$ (d) $\frac{18V}{5}$

14. Two spheres A and B of radius a and b respectively are at same electric potential. The ratio of the surface charge densities of A and B is

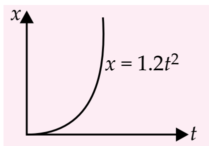
(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $\frac{a^2}{b^2}$ (d) $\frac{b^2}{a^2}$

15. If the two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin(\omega t + \pi/3)$ interfere at a point, the amplitude of the resulting wave will be approximately

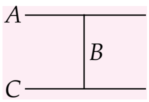
(a) 7 (b) 5 (c) 6 (d) 3.5

16. Figure shows the distance-time graph of the motion of a car. It follows from the graph that the car is

(a) at rest
(b) in uniform motion
(c) in non-uniform motion
(d) uniformly accelerated



17. In figure, all the three rods are of equal length L and same mass M. The system is rotated such that rod B is the axis of rotation. What is the moment of inertia of the system?



(a) $\frac{ML^2}{3}$ (b) $\frac{ML^2}{6}$

(c) $\frac{4}{3}ML^2$ (d) $\frac{2}{3}ML^2$

18. The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the earth is (here R is the radius of the earth)

(a) $\left(\frac{n}{n+1}\right)mgR$ (b) $\left(\frac{n}{n-1}\right)mgR$

(c) $nmgR$ (d) $\frac{mgR}{n}$

19. If Q, E and W denote respectively the heat added, change in internal energy and the work done in a closed cyclic process, then

(a) $W = 0$ (b) $Q = W = 0$
(c) $E = 0$ (d) $Q = 0$

20. A body constrained to move in the y direction is subjected to a force $\vec{F} = (2\hat{i} + 15\hat{j} + 6\hat{k})$ newton. The work done by the force in moving the body through a distance of 10 m along y-axis is

(a) 100 J (b) 150 J (c) 60 J (d) 20 J

21. The equation $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$, where the symbols carry the usual meaning and A, T and λ are positive, represents a wave of

(a) amplitude $2A$ (b) period $\frac{T}{\lambda}$

(c) speed $\frac{\lambda}{T}$

(d) velocity in negative x-direction

22. A body starting from rest moves with constant acceleration. The ratio of distance covered by the body during the 5^{th} second to that covered in 5 seconds is

(a) $\frac{9}{25}$ (b) $\frac{3}{25}$ (c) $\frac{25}{9}$ (d) $\frac{1}{25}$

23. A 40.0 kg boy is standing on a plank of mass 160 kg. The plank originally at rest, is free to slide on a smooth frozen lake. The boy walks along the plank at a constant speed of 1.5 m s^{-1} relative to the plank. The speed of the boy relative to the ice surface is

(a) 1.8 m s^{-1} (b) 1.6 m s^{-1}
(c) 1.2 m s^{-1} (d) 1.5 m s^{-1}

24. A radioactive nucleus of mass M emits a photon of frequency υ and the nucleus recoils. The recoil energy will be

(a) $Mc^2 - h\upsilon$ (b) $\frac{h^2\upsilon^2}{2Mc^2}$

(c) zero (d) $h\upsilon$

25. A drunkard walking in a narrow lane takes 8 steps forward and 6 steps backward, followed again by 8 steps forward and 6 steps backward and so on. Each step is 1 m long and requires 1 s. Determine how long the drunkard takes to fall in a pit 18 m away from the start.

(a) 18 s (b) 126 s (c) 78 s (d) 62 s

26. Which of the following statements is true?

(a) Sound waves cannot interfere.
(b) Only light waves may interfere.
(c) The de Broglie waves associated with moving particles can interfere.
(d) The Bragg formula for crystal structure is an example of the corpuscular nature of electromagnetic radiation.

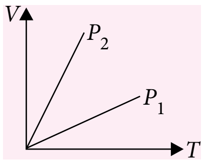
27. Which of the following physical quantity has a ratio of 10^3 between its SI units and CGS units?
(a) Universal gravitational constant
(b) Boltzmann's constant
(c) Planck's constant
(d) Young's modulus of elasticity

28. Which of the following letters do not suffer lateral inversion?
(a) HGA (b) HOX
(c) VET (d) YUL

29. If unit vectors $\hat{A}$ and $\hat{B}$ are inclined at an angle θ, then $|\hat{A}-\hat{B}|$ is
(a) $2\sin\frac{\theta}{2}$ (b) $2\cos\frac{\theta}{2}$
(c) $2\tan\frac{\theta}{2}$ (d) $\tan\theta$

30. Two liquid drops of equal radii are falling through air with the terminal velocity v. If these two drops coalesce to form a single drop, its terminal velocity will be
(a) $\sqrt{2}\,v$ (b) $2v$ (c) $\sqrt[3]{4}\,v$ (d) $\sqrt[3]{2}\,v$

31. The given figure shows the volume V versus temperature T graphs for a certain mass of a perfect gas at two constant pressures of P_1 and P_2. What inference can you draw from the graphs?



(a) $P_1 > P_2$
(b) $P_1 < P_2$
(c) $P_1 = P_2$
(d) No inference can be drawn due to insufficient information

32. If R is universal gas constant, the amount of heat needed to raise the temperature of 2 moles of an ideal monoatomic gas from 273 K to 373 K when no work is done will be
(a) $100R$ (b) $150R$ (c) $300R$ (d) $500R$

33. Two pendulums of lengths 121 cm and 100 cm start vibrating. At some instant the two are in the mean position in the same phase. After how many vibrations of the shorter pendulum, the two will be in phase in the mean position?
(a) 10 (b) 11 (c) 20 (d) 21

34. An air-tight cage with a parrot sitting in it is suspended from the spring balance. The parrot starts flying. The reading of the spring balance will
(a) increase (b) decrease
(c) not change (d) be zero.

35. There is a mine of depth about 2.0 km. In this mine the conditions as compared to those at the surface are
(a) lower air pressure, higher acceleration due to gravity
(b) higher air pressure, lower acceleration due to gravity
(c) higher air pressure, higher acceleration due to gravity
(d) lower air pressure, lower acceleration due to gravity.

36. A ball is rolled off along the edge of table (horizontal) with velocity 4 m s^{-1}. It hits the ground after time 0.4 s. Which one of the following is wrong?
(a) The height of the table is 0.8 m.
(b) It hits the ground at angle of 60° with the vertical.
(c) It covers a horizontal distance 1.6 m from the table.
(d) It hits the ground with vertical velocity 4 m s^{-1}.

37. An archaeologist analyses the wood in a prehistoric structure and finds that C^{14} (half-life = 5700 years) to C^{12} is only one-fourth of that found in the cells of buried plants. The age of the wood is about
(a) 5700 years (b) 2580 years
(c) 11,400 years (d) 22,800 years

38. Which one of the following is the property of a monochromatic, plane electromagnetic wave in free space?
(a) Electric and magnetic fields have a phase difference of $\pi/2$.
(b) The energy contribution of both electric and magnetic fields are equal.
(c) The direction of propagation is in the direction of $\vec{B}\times\vec{E}$.
(d) The pressure exerted by the wave is the product of its speed and energy density.

39. What happens to the fringe pattern if in the path of one of the slits a glass plate which absorbs 50% energy is interposed?
(a) The bright fringes become brighter and dark fringes become darker
(b) No fringes are observed
(c) The fringe width decreases
(d) None of these

40. If the electron velocity is $(2\hat{i}+3\hat{j})$ m s^{-1} and it is subjected to a magnetic field, $4\hat{k}$ T, then
(a) speed of electron will change
(b) path of electron will change
(c) Both (a) and (b)
(d) None of these

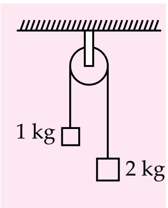
41. The magnetic field at a distance d from a short bar magnet in longitudinal and transverse positions are in the ratio
(a) 1 : 1 (b) 2 : 3 (c) 2 : 1 (d) 3 : 2

42. A resistor of 500 Ω, an inductance of 0.5 H are in series with an ac which is given by
$V = 100\sqrt{2}\sin(1000t)$. The power factor of the combination is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) 0.5 (d) 0.6

43. A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. The emergent ray is
(a) normal to the face through which it emerges
(b) inclined at 30° to the face through which it emerges
(c) inclined at 60° to the face through which it emerges
(d) None of these

44. The decreasing order of wavelength of infrared, microwaves, ultraviolet and gamma rays is
(a) microwaves, infrared, ultraviolet, gamma rays
(b) gamma rays, ultraviolet, infrared, microwaves
(c) microwaves, gamma rays, infrared, ultraviolet
(d) infrared, microwaves, ultraviolet, gamma rays

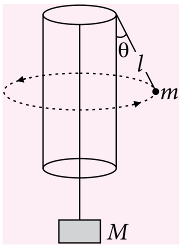
45. Two blocks of masses 1 kg and 2 kg are connected by a metal wire going over a smooth pulley as shown in figure. The breaking stress of the metal is 2×10^9 N m^{-2}. What should be the minimum radius of the wire used if it is not to break? (Take $g = 10$ m s^{-2})



(a) 4.6×10^{-5} m
(b) 4.6×10^{-6} m
(c) 2.5×10^{-6} m
(d) 2.5×10^{-5} m

46. A uniform metal chain is placed on a rough table such that one end of it hangs down over the edge of the table. When one-third of its length hangs over the edge, the chain starts sliding. Then the coefficient of static friction is
(a) 3/4 (b) 1/4 (c) 2/3 (d) 1/2

47. A large mass M and a small mass m hang at the two ends of a string that passes through a smooth tube as shown in figure. The mass m moves around a circular path in a horizontal plane. The length of the string from mass m to the top of the tube is l, and θ is the angle the string makes with the vertical. What should be the frequency (υ) of rotation of mass m so that mass M remains stationary?



(a) $\upsilon = \frac{1}{2\pi}\sqrt{\frac{mg}{M}}$

(b) $\upsilon = \frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$

(c) $\upsilon = \frac{1}{\pi}\sqrt{\frac{Mg}{m}}$

(d) $\upsilon = \frac{1}{2\pi}\frac{M}{m}\sqrt{\frac{g}{l}}$

48. The moment of inertia of a sphere of mass M and radius R about an axis passing through its centre is $\frac{2}{5}MR^2$. The radius of gyration of the sphere about a parallel axis to the above and tangent to the sphere is
(a) $\frac{7}{5}R$ (b) $\frac{3}{5}R$ (c) $\left(\sqrt{\frac{7}{5}}\right)R$ (d) $\left(\sqrt{\frac{3}{5}}\right)R$

49. A lift is tied with thick iron wires and its mass is 1000 kg. The minimum diameter of wire if the maximum acceleration of lift is 1.2 m s^{-2} and the maximum safe stress is 1.4×10^8 N m^{-2} is ($g = 10$ m s^{-2})
(a) 0.00141 m (b) 0.00282 m
(c) 0.005 m (d) 0.01 m

50. When an AC voltage, of variable frequency is applied to series L-C-R circuit, the current in the circuit is the same at 4 kHz and 9 kHz. The current in the circuit is maximum at
(a) 5 kHz (b) 6.5 kHz
(c) 4.2 kHz (d) 6 kHz

51. A concave mirror of focal length f (in air) is immersed in water (μ = 4/3). The focal length of the mirror in water will be
(a) f (b) $\frac{4}{3}f$ (c) $\frac{3}{4}f$ (d) $\frac{7}{3}f$

52. In Young's double slit experiment, when two light waves form third minimum intensity, they have
(a) phase difference of 3π
(b) phase difference of $\frac{5\pi}{2}$
(c) path difference of 3λ
(d) path difference of $\frac{5\lambda}{2}$

53. An LC resonant circuit contains a 400 pF capacitor and a 100 μH inductor. It is set into oscillation coupled to an antenna. The wavelength of the radiated electromagnetic waves is

(a) 377 mm (b) 377 m
(c) 377 cm (d) 3.77 cm

54. If the wavelength of the first line of the Balmer series of hydrogen atom is 6561 Å, the wavelength of the second line of the series should be

(a) 13122 Å (b) 3280 Å
(c) 4860 Å (d) 2187 Å

55. A p-type semiconductor has acceptor level 57 meV above the valence band. The maximum wavelength of light required to create hole is

(a) 57 Å (b) 57×10^{-3} Å
(c) 217100 Å (d) 11.61×10^{-33} Å

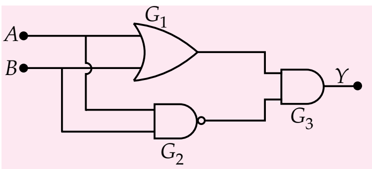
56. A particle is executing simple harmonic motion with an amplitude A and time period T. The displacement of the particle after $2T$ period from its initial position is

(a) A (b) $4A$ (c) $8A$ (d) zero

57. A thin disc having radius r and charge q distributed uniformly over the disc is rotated with υ rotations per second about its axis. The magnetic field at the centre of the disc is

(a) $\frac{\mu_0 q\upsilon}{2r}$ (b) $\frac{\mu_0 q\upsilon}{r}$ (c) $\frac{\mu_0 q\upsilon}{4r}$ (d) $\frac{3\mu_0 q\upsilon}{4r}$

58. The following configuration of gates is equivalent to



(a) NAND
(b) XOR
(c) OR
(d) AND

59. At a given temperature, velocity of sound in oxygen and in hydrogen has the ratio

(a) 4 : 1 (b) 1 : 4 (c) 1 : 1 (d) 2 : 1

60. In the Davisson and Germer experiment, the velocity of electrons emitted from the electron gun can be increased by

(a) increasing the potential difference between the anode and filament
(b) increasing the filament current
(c) decreasing the filament current
(d) decreasing the potential difference between the anode and filament

SOLUTIONS

1. **(b):** Given : $m = 0.5$ kg, $v = kx^{3/2}$
where, $k = 5\ \text{m}^{-1/2}\ \text{s}^{-1}$

Acceleration, $a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ $\left(\because v = \frac{dx}{dt}\right)$

As $v^2 = k^2x^3$

Differentiating both sides with respect to x, we get

$2v\frac{dv}{dx} = 3k^2x^2$ $\therefore$ Acceleration, $a = \frac{3}{2}k^2x^2$

Force, $F = \text{Mass} \times \text{Acceleration} = \frac{3}{2}mk^2x^2$

Work done, $W = \int F dx = \int_0^2 \frac{3}{2}mk^2x^2 dx$

$$W = \frac{3}{2}mk^2\left[\frac{x^3}{3}\right]_0^2 = \frac{3}{6} \times 0.5 \times 5^2 \times [2^3 - 0] = 50\ \text{J}$$

2. **(b):** Here, $\upsilon_1 = 2.8$ rps, $\upsilon_2 = ?$, $I_2 = 0.7I_1$

Using conservation of angular momentum,

$I_1\omega_1 = I_2\omega_2$

$\frac{\omega_2}{\omega_1} = \frac{I_1}{I_2} = \frac{1}{0.7}$ $\therefore$ $\frac{\upsilon_2}{\upsilon_1} = \frac{10}{7}$

$\upsilon_2 = \frac{10}{7}\upsilon_1 = \frac{10}{7} \times 2.8 = 4.0$ rps

3. **(a):** Applying principle of conservation of linear momentum, we get

$$m \times 0 + \frac{m}{20} \times v = \left(m + \frac{m}{20}\right)v' = \frac{21}{20}mv' \Rightarrow v' = \frac{v}{21}$$

4. **(b):** The relative velocity of rain with respect to car is inclined to the vertical in the backward direction. Therefore, it will strike the front screen.

5. **(c):** Error in weighing = difference in weight at two different heights

$$= mg\left[1 - \frac{2h_1}{R}\right] - mg\left[1 - \frac{2h_2}{R}\right]$$

$$= \frac{2mg}{R}(h_2 - h_1) = \frac{2m}{R} \times \frac{GM}{R^2} \times \frac{h}{2}$$

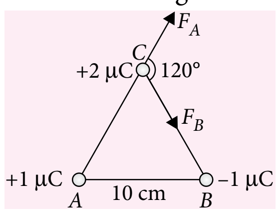
[where $h_2 - h_1 = h/2$]

$$= \frac{2m}{R^3} \times G \times \frac{4}{3}\pi R^3 \rho \times \frac{h}{2} = \frac{4}{3}\pi G m \rho h$$

6. **(b):** F_A = Force on charge at C due to charge at A

$$= 9 \times 10^9 \times \frac{10^{-6} \times 2 \times 10^{-6}}{(10 \times 10^{-2})^2}$$

$= 1.8$ N



F_B = Force on charge at C due to charge at B

$$= 9\times10^9 \times \frac{10^{-6}\times2\times10^{-6}}{(0.1)^2} = 1.8\text{ N}$$

Net force on charge at C,

$$F = \sqrt{F_A^2 + F_B^2 + 2F_AF_B\cos120°}$$

$$= \sqrt{(1.8)^2 + (1.8)^2 + 2(1.8)(1.8)(-1/2)} = 1.8\text{ N}$$

7. (a) : Resistance of wire,

$$R = \rho\frac{l}{A} = \frac{\rho l^2}{V} \Rightarrow R \propto l^2$$

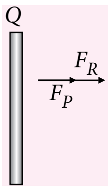
$$\therefore \frac{R_1}{R_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{l}{2l}\right)^2 \text{ or } R_2 = 4R_1$$

$\therefore$ Change in resistance,

$$\frac{\Delta R}{R}\times100 = \frac{R_2 - R_1}{R_1}\times100 = \frac{4R_1 - R_1}{R_1}\times100$$

$$= 300\%$$

8. (c) : Force on wire Q due to wire R



$$F_R = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{2\times20\times10}{0.02}\times0.1$$

$= 20 \times 10^{-5}$ N (towards right)

Force on wire Q due to wire P

$$F_P = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{2\times30\times10}{0.1}\times0.1$$

$= 6 \times 10^{-5}$ N (towards right)

Net force on Q

$F = F_R + F_P = 20 \times 10^{-5} + 6 \times 10^{-5}$

$= 26 \times 10^{-5}$ N

$= 2.6 \times 10^{-4}$ N (towards right)

9. (d): $\phi = NAB = (25)(\pi)(1.8 \times 10^{-2})^2\,[0.2t - 0.05t^2]$

$= 0.025(0.2t - 0.05t^2)$

$$\varepsilon = \left|\frac{d\phi}{dt}\right| = |0.025(0.2 - 0.1t)|$$

At $t = 3$ s, $\varepsilon = 0.0025$ V

$$I = \frac{\varepsilon}{R} = 0.0016\text{ A}$$

$\therefore$ $P = \varepsilon I = 4 \times 10^{-6}$ W $= 4\ \mu$W

10. (c) : Light should fall normally on the silvered face.

$r_2 = 0° \therefore r_1 = A = 30°$

$$\text{Now, } \mu = \frac{\sin i_1}{\sin r_1} \text{ or } \sqrt{2} = \frac{\sin i_1}{\sin 30°}$$

This gives $i_1 = 45°$

11. (b): For K_α line, $\lambda_{K_\alpha} \propto \frac{1}{(Z-1)^2}$

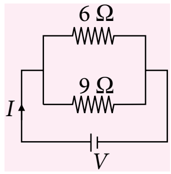
$$\text{So, } \frac{\lambda_2}{\lambda_1} = \left(\frac{Z_1 - 1}{Z_2 - 1}\right)^2$$

$$\Rightarrow \frac{\lambda_2}{\lambda} = \left(\frac{43-1}{29-1}\right)^2 = \left(\frac{42}{28}\right)^2 \Rightarrow \lambda_2 = \frac{9}{4}\lambda$$

12. (b): In reverse biasing of p-n junction diode, the applied reverse voltage establishes an electric field which acts in the same direction as the electric field in the potential barrier. Therefore, the height of potential barrier is increased.

13. (b): Given circuit is balanced Wheatstone bridge and hence no current flows through 4 Ω resistance connected across diagonal.

So, equivalent resistance of circuit,



$$R_{eq} = \frac{6\times9}{6+9} = \frac{18}{5}\,\Omega$$

$$\therefore \text{ Current, } I = \frac{V}{R} = \frac{V}{18/5} = \frac{5V}{18}\text{A}$$

14. (b) **15. (c)**

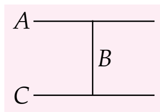
16. (d): $x = 1.2t^2$,

$$\text{Velocity, } v = \frac{dx}{dt} = \frac{d}{dt}(1.2t^2) = 2.4t$$

$$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}(2.4t) = 2.4 = \text{a constant}$$

Thus the given motion is uniformly accelerated.

17. (b): As is clear from figure, rod B passes through centre of rods A and C. Therefore, moment of inertia of the system about rod B is



$$I = I_A + I_B + I_C$$

$$I = \frac{ML^2}{12} + 0 + \frac{ML^2}{12} = \frac{ML^2}{6}$$

18. (a) : Gravitational potential energy of mass m at any point at a distance r from the centre of earth is

$$U = -\frac{GMm}{r}$$

At the surface of earth $r = R$,

$$\therefore U_s = -\frac{GMm}{R} = -mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

At the height $h = nR$ from the surface of earth

$r = R + h = R + nR = R(1 + n)$

$$\therefore U_h = -\frac{GMm}{R(1+n)} = -\frac{mgR}{(1+n)}$$

Change in gravitational potential energy is

$$\Delta U = U_h - U_s = -\frac{mgR}{(1+n)} - (-mgR)$$

$$= -\frac{mgR}{1+n} + mgR = mgR\left(1-\frac{1}{1+n}\right) = mgR\left(\frac{n}{1+n}\right)$$

19. (c) : In a closed cyclic process, the system returns to its initial state. Therefore, the change in internal energy is zero, *i.e.*, $E = 0$.

20. (b): Here, $\vec{F} = (2\hat{i} + 15\hat{j} + 6\hat{k})$ N

and $\vec{r} = 10\hat{j}$ m

$\therefore \quad W = \vec{F}\cdot\vec{r} = (2\hat{i}+15\hat{j}+6\hat{k})\cdot 10\hat{j} = 150$ J

21. (c) : $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$

Speed of wave $= \frac{\lambda}{T}$

Amplitude = A

Period = T,

Velocity is along +x-direction.

22. (a) : Distance covered in 5^{th} second is

$D_5 = 0 + \frac{a}{2}(2\times 5 - 1) = \frac{9a}{2} \quad (\because \ u = 0)$

Distance covered in 5 seconds is

$S_5 = 0 + \frac{1}{2}a\times 5^2 = \frac{25a}{2} \qquad (\because \ u = 0)$

$\therefore \quad \frac{D_5}{S_5} = \frac{9}{25}$

23. (c) : The system is not subjected to any external force and hence conservation of momentum can be used. Let m_b and m_p represent the masses of the boy and the plank respectively. Let v_{bi}, v_{pi} and v_{bp} be the velocity of the boy with respect to ice, that of the plank with respect to ice and that of the boy with respect to the plank respectively. Then,

$m_b v_{bi} + m_p v_{pi} = 0$ ...(i)

$v_{bi} = v_{bp} + v_{pi}$ ...(ii)

or $\quad v_{pi} = v_{bi} - v_{bp}$

Substituting the value of v_{pi} in eq. (i), we get

$m_b v_{bi} + m_p(v_{bi} - v_{bp}) = 0$

or $\quad v_{bi}(m_b + m_p) = m_p v_{bp}$ or $\quad v_{bi} = \frac{m_p v_{bp}}{m_b + m_p}$

Substituting the given values, we get

$$v_{bi} = \frac{160\text{ kg}\times 1.5\text{ m s}^{-1}}{(40+160)\text{ kg}} = \frac{240}{200}\text{ m s}^{-1} = 1.2\text{ m s}^{-1}$$

24. (b): Momentum of emitted photon, $p_{photon} = \frac{h\upsilon}{c}$

Let v be the recoil speed of nucleus.

According to law of conservation of momentum,

$p_{nucleus} = p_{photon}$

$\therefore \quad Mv = \frac{h\upsilon}{c}$ or $v = \frac{h\upsilon}{Mc}$

The recoil energy of the nucleus

$$= \frac{1}{2}Mv^2 = \frac{1}{2}M\left(\frac{h\upsilon}{Mc}\right)^2 = \frac{h^2\upsilon^2}{2Mc^2}$$

25. (c) : When the drunkard walks 8 steps forward and 6 steps backward, the displacement in the first 14 steps = 8 m – 6 m = 2 m

Time taken for first 14 steps = 14 s

Time taken by drunkard to cover first 10 m of journey $= \frac{14}{2}\times 10 = 70$ s

If the drunkard takes 8 steps more, he will fall into the pit, so the time taken by the last 8 steps = 8 s

Total time taken = 70 s + 8 s = 78 s

26. (c) : Both sound and light waves exhibit the phenomenon of interference.

The Bragg formula for crystal structure is an example of the wave nature of electromagnetic radiation.

27. (a) : The value of universal gravitational constant in CGS system is 6.67×10^{-8} dyne cm^2 $g^{-2}$and in SI system is 6.67×10^{-11} N m^2 kg^{-2}.

Their corresponding ratio is

$$\frac{G\text{ in CGS unit}}{G\text{ in SI unit}} = \frac{6.67\times 10^{-8}}{6.67\times 10^{-11}} = 10^3$$

28. (b)

29. (a) : $|\hat{A} - \hat{B}|^2 = (\hat{A} - \hat{B})\cdot(\hat{A} - \hat{B})$

$= \hat{A}\cdot\hat{A} - \hat{A}\cdot\hat{B} - \hat{B}\cdot\hat{A} + \hat{B}\cdot\hat{B}$

$= 1 - \hat{A}\cdot\hat{B} - \hat{A}\cdot\hat{B} + 1 = 2 - 2\cos\theta = 2[1 - \cos\theta]$

$= 2\left[1 - 1 + 2\sin^2\frac{\theta}{2}\right] = 4\sin^2\frac{\theta}{2}$

or $\quad |\hat{A} - \hat{B}| = 2\sin\frac{\theta}{2}$

30. (c) : Let R be the radius of big drop formed and r be radius of each small drop. Then

$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3$ or $R = 2^{1/3}r$

Terminal velocity $\propto r^2$

$\therefore \quad \frac{v'}{v} = \frac{R^2}{r^2} = \left(\frac{2^{1/3}r}{r}\right)^2 = \sqrt[3]{4}$ or $v' = \sqrt[3]{4}\,v$

31. (a) : For a given T, $V \propto \frac{1}{P}$ $\therefore \quad \frac{V_2}{V_1} = \frac{P_1}{P_2}$

For a given T, $V_2 > V_1$, so $P_1 > P_2$.

32. (c) : As no work is done,

$\therefore \quad dQ = dU + dW = dU = nC_V\,dT$

$$= n\left(\frac{R}{\gamma - 1}\right)dT = 2\times\frac{R}{\left(\frac{5}{3}-1\right)}\times 100 = 300R$$

33. (b): $T_1 = 2\pi\sqrt{\frac{121}{g}}$ and $T_2 = 2\pi\sqrt{\frac{100}{g}}$

So, $T_1 > T_2$. Let the shorter pendulum makes n vibrations, then the longer pendulum will make one less than n vibrations to come in phase again.

So, $nT_2 = (n - 1)T_1$

or $\quad n\times 2\pi\sqrt{\frac{100}{g}} = (n-1)\times 2\pi\sqrt{\frac{121}{g}}$

or $\quad 10n = (n - 1)11$ or $n = 11$

34. (c) : The system is closed. The weight of parrot is supported by the air pushed downwards which in turn presses the base of cage with a weight equal to weight of parrot. These are internal forces. So there will be no change in the reading of the spring balance.

35. (b): Below the sea level the pressure is increasing with depth due to presence of atmospheric air there. The acceleration due to gravity below the surface of the earth decreases with the distance from the surface of the earth, as $g' = g\left(1 - \frac{d}{R}\right)$.

36. (b): Taking vertical downward motion,

$$s = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 0.4^2 = 0.8 \text{ m}$$

Vertical velocity on ground,

$v_y = gt = 10 \times 0.4 = 4 \text{ m s}^{-1}$

Horizontal range $= v_x t = 4 \times 0.4 = 1.6$ m

If θ is the angle at which the ball hits the ground with the vertical, then $\tan\theta = \frac{v_x}{v_y} = \frac{4}{4} = 1$

or $\quad \theta = 45°$

37. (c) : $\frac{N_0}{N} = \frac{1}{4} = \left(\frac{1}{2}\right)^{t/5700} \quad \therefore \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/5700}$

$\therefore \frac{t}{5700} = 2$ or $t = 11{,}400$ years

38. (b): The energy in EM waves is divided equally between the electric and magnetic fields.

39. (d): In normal Young's double slit apparatus,

$I_1 = I_2 = I_0$

$\therefore \quad I_{max} = \left(\sqrt{I_1} + \sqrt{I_1}\right)^2 = 4I_0$

and $I_{min} = \left(\sqrt{I_1} - \sqrt{I_1}\right)^2 = 0$

When one of the slits is covered by a glass plate $I_1 < I_0$, $I_2 = I_0$ $\therefore$ $I_{max} < 4I_0$ and $I_{min} > 0$

40. (b): Velocity is in x-y plane and magnetic field is along z-axis. Therefore, path of the electron will be a circle. Magnetic force cannot change the speed of a particle.

41. (c) : For longitudinal positions, $B_1 \propto \frac{2M}{d^3}$

For transverse positions,

$$B_2 \propto \frac{M}{d^3} \quad \therefore \quad \frac{B_1}{B_2} = 2:1$$

42. (a) : Here, $R = 500\ \Omega$, $L = 0.5$ H

Compare $V = 100\sqrt{2}\sin(1000t)$ with $V = V_0\sin\omega t$,

we get $\omega = 1000$

The inductive reactance is

$X_L = \omega L = (1000)(0.5) = 500\ \Omega$

Impedance of the RL circuit is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(500\ \Omega)^2 + (500\ \Omega)^2} = 500\sqrt{2}\ \Omega$$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{500\ \Omega}{500\sqrt{2}\ \Omega} = \frac{1}{\sqrt{2}}$

43. (a) : Angle of deviation of ray in prism is given by

$d = i + e - A$

So, $\quad e = d + A - i = 30° + 30° - 60° = 0°$

So, emergent ray will be perpendicular to face.

Or emergent ray will make an angle of 90° with the face through which it emerges.

44. (a) : The decreasing order of wavelength of the given electromagnetic waves is as follows:

$$\lambda_{\text{Microwave}} > \lambda_{\text{Infrared}} > \lambda_{\text{Ultraviolet}} > \lambda_{\text{Gamma rays}}$$

45. (a) : Here, $m_1 = 1$ kg, $m_2 = 2$ kg

Breaking stress $= 2 \times 10^9$ N m^{-2}

The tension in the string is

$$T = \frac{2m_1 m_2}{m_1 + m_2}g = \frac{2\times 1 \text{ kg}\times 2 \text{ kg}\times 10 \text{ m s}^{-2}}{(1+2)\text{ kg}} = \frac{40}{3}\text{ N}$$

If r is the minimum radius, then

$$\text{Breaking stress} = \frac{(40/3)}{\pi r^2}$$

$$r^2 = \frac{40}{3\times\pi\times 2\times 10^9} \quad \text{or} \quad r^2 = \frac{2}{3\pi}\times 10^{-8}$$

$r = 0.46 \times 10^{-4}$ m $= 4.6 \times 10^{-5}$ m

46. (d): The chain starts sliding, when

applied force = force of friction

(due to hanging part) (between chain and table)

$$\frac{1}{3}mg = f = \mu R = \mu\left(\frac{2}{3}mg\right);\ \mu = \frac{1}{2}$$

47. (b) : Tension in the string $T = mg$.
Centripetal force on the body = $mr\omega^2 = mr(2\pi\upsilon)^2$.
This is provided by the component of tension acting horizontally
i.e. $T\sin\theta = Mg\sin\theta$

$$\therefore \quad mr(2\pi\upsilon)^2 = Mg\sin\theta = \frac{Mgr}{l} \text{ or } \upsilon = \frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$$

48. (c) : Given, $I = \frac{2}{5}MR^2$
Using the theorem of parallel axes, moment of inertia of the sphere about a parallel axis tangent to the sphere is

$$I' = I + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

$$\therefore \quad I' = MK^2 = \frac{7}{5}MR^2 \Rightarrow K = \left(\sqrt{\frac{7}{5}}\right)R$$

49. (d) : When the lift is accelerated upwards with acceleration a, the tension in the rope is
$T = m(g + a) = 1000(9.8 + 1.2) = 11000$ N

$$\text{Now, stress} = \frac{F}{A} = \frac{T}{\pi r^2} \text{ or } r^2 = \frac{T}{\pi \times \text{stress}}$$

$$= \frac{11000\times 7}{22\times 1.4\times 10^8} = \frac{1}{4\times 10^4}$$

$$\text{or} \quad r = \frac{1}{200};\ \text{so } D = 2r = \frac{1}{100} = 0.01 \text{ m}$$

50. (d)

51. (a) : On immersing a mirror in water, focal length of the mirror remains unchanged.

52. (d) : For minima, path difference should be an odd multiple of half wavelength.

$$\text{or} \quad \Delta = (2n-1)\frac{\lambda}{2}$$

For third minima,

$$n = 3,\ \Delta = (2\times 3 - 1)\frac{\lambda}{2} = \frac{5\lambda}{2}$$

53. (b) : Frequency of LC oscillations is

$$\upsilon = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{100\times 10^{-6}\times 400\times 10^{-12}}}$$

$$= 0.0796 \times 10^7 \text{ Hz}$$

$$\text{So, wavelength, } \lambda = \frac{c}{\upsilon} = \frac{3\times 10^8}{0.0796\times 10^7} = 377 \text{ m}$$

54. (c) : Wavelength of spectral line in Balmer series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

For first line of Balmer series, $n = 3$

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

For second line of Balmer series, $n = 4$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16} \quad \therefore \quad \frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$$\therefore \quad \lambda_2 = \frac{20}{27}\times 6561 = 4860 \text{ Å}$$

55. (c) : $\lambda(\text{in Å}) = \dfrac{12375}{E \text{ (in eV)}} = \dfrac{12375}{57\times 10^{-3}} \text{ Å} \approx 217100 \text{ Å}$

56. (d) : The particle completes one oscillation in time T. Therefore, in time $2T$, it will complete two oscillations and will reach to its starting point, *i.e.*, initial position. Therefore, the displacement is zero.

57. (b)

58. (b) : Output of $G_1 = (A + B)$
Output of $G_2 = \overline{A\cdot B}$
Output of G_3 is

$$Y = (A+B)\cdot(\overline{A\cdot B}) = (A+B)\cdot(\bar{A}+\bar{B})$$

$$= A\cdot\bar{A} + A\cdot\bar{B} + B\cdot\bar{A} + B\cdot\bar{B} = A\cdot\bar{B} + \bar{A}\cdot B$$

It is the Boolean expression of XOR gate.
Hence, the given configuration of gates is equivalent to XOR gate.

59. (b) : Speed of sound in gas is

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where the symbols have their usual meanings.
Since both oxygen and hydrogen are diatomic gases and at same temperature, the ratio of velocities will be

$$\frac{v_{O_2}}{v_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

60. (a) : As $eV = \frac{1}{2}mv^2$ or $v \propto \sqrt{V}$
Thus velocity of electron emitted from electron gun can be increased by increasing the potential difference between anode and filament in Davisson and Germer experiment.

❖❖

PHYSICS MUSING

SOLUTION SET-21

1. **(b) :** $F = \sqrt{F_T^2 + F_R^2}$

$F_T = m\alpha\frac{l}{2}$ and $F_R = m\omega^2\frac{l}{2} = m\left(2\alpha \times \frac{\pi}{2}\right)\frac{l}{2}$

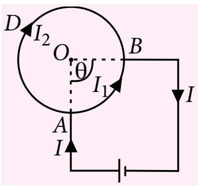
[Using $\omega^2 = \omega_0^2 + 2\alpha\theta$]

$\therefore \quad F = m\alpha\frac{l}{2}\sqrt{1+\pi^2}$

2. **(d) :** Magnetic induction at the centre of an arc carrying current I is

$$B = \frac{\mu_0 I}{4\pi r}\theta \quad \text{or} \quad B \propto I\theta$$

The total current I is divided along two arcs such that



$$I \propto \frac{1}{\text{Resistance of arc}}$$

or $\quad I \propto \frac{1}{\text{Length of arc}}$

or $\quad I \propto \frac{1}{\text{Angle subtended at centre } (\theta)}$

$\therefore \quad I\theta = \text{constant}$

or $\quad I_1\theta = I_2(2\pi - \theta)$

Hence the magnetic fields at the centre due to arcs *ACB* and *ADB* are equal and opposite. The net magnetic induction at the centre *O* is zero.

3. **(a) :** For first fragment of particle, $ut_1 - \frac{1}{2}gt_1^2 = -H$

$\frac{1}{2}gt_1^2 - ut_1 - H = 0$

$$t_1 = \frac{u \pm \sqrt{u^2 + 2gH}}{g} = \frac{u + \sqrt{u^2 + 2gH}}{g}$$

For second fragment of particle, along the vertical direction,

$$\frac{1}{2}gt_2^2 + ut_2 - H = 0$$

$$t_2 = \frac{-u \pm \sqrt{u^2 + 2gH}}{g} = \frac{-u + \sqrt{u^2 + 2gH}}{g}$$

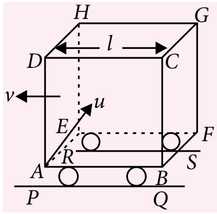
$\therefore \quad \Delta t = t_1 - t_2 = 2u/g$

4. **(a) :** Let v is the velocity of carriage with respect to ground and u is the velocity of shot with respect to carriage as shown in the figure.

$$\frac{u^2 \sin(2 \times 45°)}{g} = l\sqrt{2} \Rightarrow u = \sqrt{gl\sqrt{2}} \qquad \text{...(i)}$$

Using momentum conservation principle

$Mv = m[u(\cos 45°) \times \cos 45° - v]$



$\Rightarrow \; (M+m)v = \frac{mu}{2}$

$\Rightarrow \; v = \frac{mu}{2(M+m)} = \frac{m\sqrt{gl\sqrt{2}}}{2(M+m)}$

5. **(d) :** $F = pt - qx$

Acceleration of the particle,

$$a = \frac{F}{m} = \frac{1}{m}(pt - qx) \Rightarrow \frac{da}{dt} = \frac{p - qv}{m}$$

$$\Rightarrow \frac{d^2a}{dt^2} = -\frac{q}{m}a = -\omega^2 a \Rightarrow \omega = \sqrt{\frac{q}{m}}$$

Hence, the acceleration of particle varies sinusoidally with time.

6. **(a) :** Young's modulus, $Y = \frac{F}{A}.\frac{L}{l}$

Force constant, $k = \frac{F}{l} = \frac{YA}{L}$

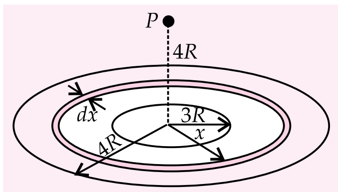
where l is the extension in the rod of original length L and cross-sectional area A when a force $F = Mg$ is applied. Now, the time period of vertical oscillations is given by

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{ML}{YA}} \quad \therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{3}{2}}$$

7. **(a) :** Mass per unit area of the disc,

$$\sigma = \frac{\text{Mass}}{\text{Area}} = \frac{M}{\pi\left((4R)^2 - (3R)^2\right)} = \frac{M}{7\pi R^2}$$

Consider a ring of radius x and thickness dx as shown in the figure.



Mass of the ring, $dM = \sigma 2\pi x dx$

$$= \frac{2\pi M x dx}{7\pi R^2}$$

Potential at point P due to annular disc is

$$V_P = \int_{3R}^{4R} -\frac{G dM}{\sqrt{(4R)^2 + (x)^2}} = -\frac{GM2\pi}{7\pi R^2}\int_{3R}^{4R}\frac{x dx}{\sqrt{16R^2 + x^2}}$$

Solving, we get

$$V_P = -\frac{GM2\pi}{7\pi R^2}\left[\sqrt{16R^2 + x^2}\right]_{3R}^{4R}$$

$$= -\frac{2GM}{7R}\left(4\sqrt{2} - 5\right)$$

Work done in moving a unit mass from P to ∞

$= V_\infty - V_P$

$$= 0 - \left(\frac{-2GM}{7R}(4\sqrt{2}-5)\right) = \frac{2GM}{7R}(4\sqrt{2}-5)$$

8. **(a) :** $\Delta Q = -\Delta U;\ nCdT = -nC_V dT$

$C = -C_V = -R/(\gamma - 1)$

$dQ = dU + dW;\ 2dQ = dW$

$$2CdT = \frac{RT}{V}dV$$

$\Rightarrow TV^{(\gamma-1)/2} = \text{constant}$

and $TV^{1/5} = \text{constant}$ (given)

$$\text{So}\quad \frac{\gamma-1}{2} = \frac{1}{5} \quad\Rightarrow\quad \gamma = \frac{7}{5}$$

9. **(c) :** Net magnetic field at point P,

$$\vec{B} = \frac{\mu_0 I}{4\pi R}\left(\frac{3\pi}{2}\right)\odot + \frac{\mu_0 I}{4\pi R}\otimes + \frac{\mu_0 I}{4\pi R}\otimes$$

$$\vec{B} = \frac{\mu_0 I}{8\pi R}(3\pi)\odot - \frac{\mu_0 I}{2\pi R}\odot$$

$$= \frac{\mu_0 I}{8\pi R}(3\pi - 4)\odot$$

10. **(c) :** $f_{1l} = \mu_1 m_1 g = 0.4 \times 2 \times 10 = 8$ N

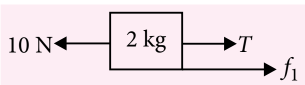
$f_{2l} = \mu_2 m_2 g = 0.6 \times 4 \times 10 = 24$ N

$f_l > F_{applied}$

So, system will not move i.e. it will be in equilibrium

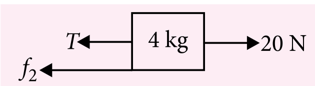
For equilibrium of 2 kg block

$10 = 8 + T \Rightarrow T = 2$ N



For equilibrium of 4 kg block

$T + f_2 = 20 \Rightarrow f_2 = 18$ N



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JEE MAIN

SOLVED PAPER 2015

1. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to

(a) $\frac{5h}{8}$ (b) $\frac{3h^2}{8R}$ (c) $\frac{h^2}{4R}$ (d) $\frac{3h}{4}$

2. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is

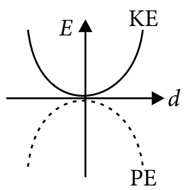
(a) 5.48 V/m (b) 7.75 V/m
(c) 1.73 V/m (d) 2.45 V/m

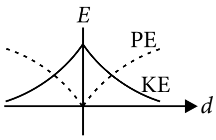
3. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to
(g = gravitational acceleration)

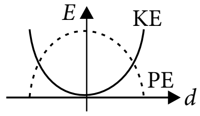
(a) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (b) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

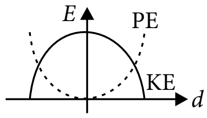
(c) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

4. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?
(graphs are schematic and not drawn to scale)

(a) 

(b) 

(c) 

(d) 

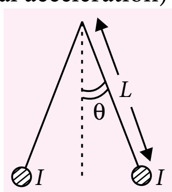
5. A train is moving on a straight track with speed 20 m s^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 m s^{-1}) close to

(a) 18% (b) 24%
(c) 6% (d) 12%

6. When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} m s^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to

(a) $1.6 \times 10^{-6}\ \Omega$ m (b) $1.6 \times 10^{-5}\ \Omega$ m
(c) $1.6 \times 10^{-8}\ \Omega$ m (d) $1.6 \times 10^{-7}\ \Omega$ m

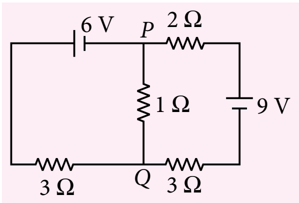
7. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then the value of I is
(g = gravitational acceleration)



(a) $2\sqrt{\frac{\pi g L}{\mu_0}}\tan\theta$ (b) $\sqrt{\frac{\pi\lambda g L}{\mu_0}}\tan\theta$

(c) $\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$ (d) $2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$

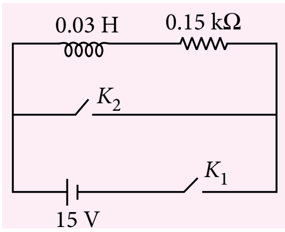
8. In the circuit shown, the current in the 1 Ω resistor is



(a) 0.13 A, from Q to P
(b) 0.13 A, from P to Q
(c) 0.3 A, from P to Q
(d) 0 A

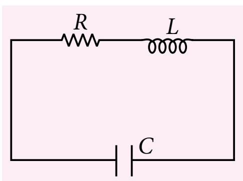
9. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is
(a) 100 μm (b) 300 μm
(c) 1 μm (d) 30 μm

10. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be ($e^5 \cong 150$)

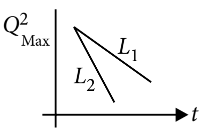


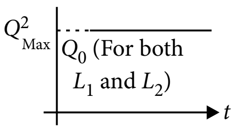
(a) 6.7 mA (b) 0.67 mA
(c) 100 mA (d) 67 mA

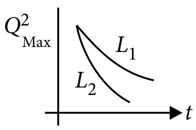
11. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown here.

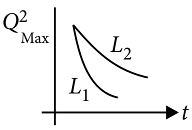


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly ? (plots are schematic and not drawn to scale)

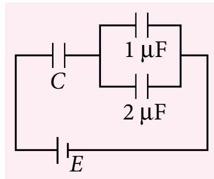
(a) 

(b) 

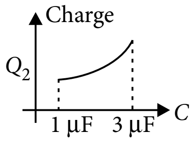
(c) 

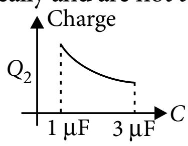
(d) 

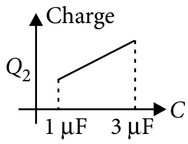
12. In the given circuit, charge Q_2 on the 2 μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of 'C' is given properly by

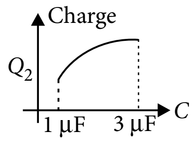


(figures are drawn schematically and are not to scale)

(a) 

(b) 

(c) 

(d) 

13. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is

(a) $\dfrac{4MR^2}{9\sqrt{3}\pi}$ (b) $\dfrac{4MR^2}{3\sqrt{3}\pi}$

(c) $\dfrac{MR^2}{32\sqrt{2}\pi}$ (d) $\dfrac{MR^2}{16\sqrt{2}\pi}$

14. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\dfrac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of g is
(a) 1% (b) 5% (c) 2% (d) 3%

15. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam
(a) bends downwards
(b) bends upwards
(c) becomes narrower
(d) goes horizontally without any deflection

16. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are
(a) 2005 kHz, 2000 kHz and 1995 kHz
(b) 2000 kHz and 1995 kHz
(c) 2 MHz only
(d) 2005 kHz and 1995 kHz

17. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

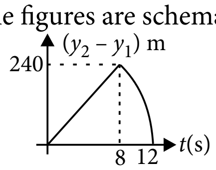
In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is

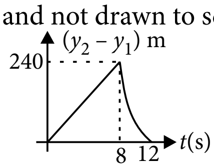
(a) $ln2, 2ln2$ (b) $2ln2, 8ln2$
(c) $ln2, 4ln2$ (d) $ln2, ln2$

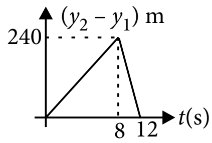
18. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is

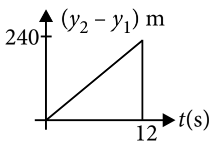
(a) $T \propto \frac{1}{R}$ (b) $T \propto \frac{1}{R^3}$
(c) $T \propto e^{-R}$ (d) $T \propto e^{-3R}$

19. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10$ m/s^2)
(The figures are schematic and not drawn to scale)

(a)


(b)


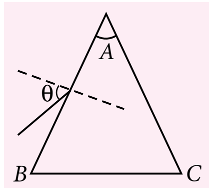
(c)


(d)


20. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(a) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(b) $2R < R_4$
(c) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(d) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

21. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided



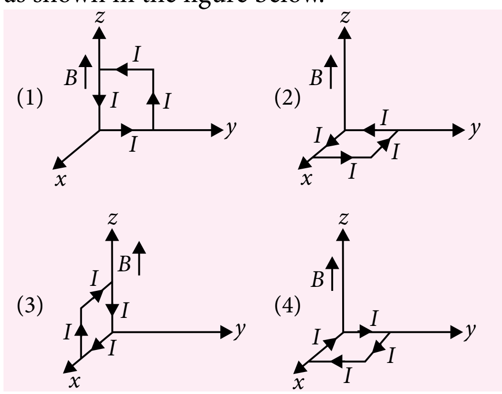
(a) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(b) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(c) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(d) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

22. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figure below.



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium.

(a) (2) and (4), respectively
(b) (2) and (3), respectively
(c) (1) and (2), respectively
(d) (1) and (3), respectively

23. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then

(a) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$

(b) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$

(c) $\vec{F}_1 = \vec{F}_2 = 0$

(d) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$ is radially outwards

24. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(a) 56% (b) 62% (c) 44% (d) 50%

25. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is

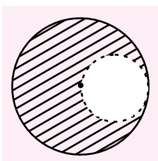
$\left(\gamma = \frac{C_p}{C_v}\right)$

(a) $\frac{\gamma+1}{2}$ (b) $\frac{\gamma-1}{2}$

(c) $\frac{3\gamma+5}{6}$ (d) $\frac{3\gamma-5}{6}$

26. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is

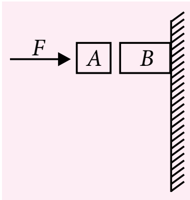
(G = gravitational constant)



(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$

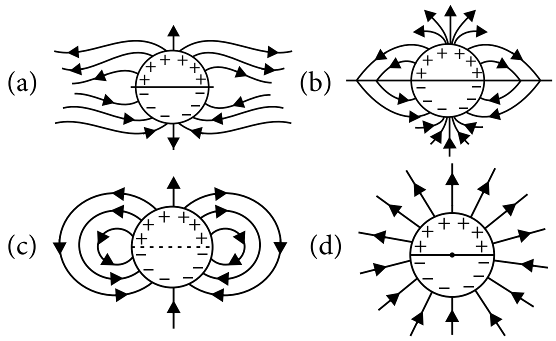
(c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

27. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is



(a) 120 N (b) 150 N (c) 100 N (d) 80 N

28. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (Figures are schematic and not drawn to scale)



29. As an electron makes a transition from an excited state to the ground state of a hydrogen -like atom/ion

(a) kinetic energy decreases, potential energy increases but total energy remains same

(b) kinetic energy and total energy decrease but potential energy increases

(c) its kinetic energy increases but potential energy and total energy decrease

(d) kinetic energy, potential energy and total energy decrease

30. Match List-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list:

List-I		List-II	
P.	Franck-Hertz Experiment.	(i)	Particle nature of light
Q.	Photo-electric Experiment.	(ii)	Discrete energy levels of atom
R.	Davison-Germer Experiment.	(iii)	Wave nature of electron
		(iv)	Structure of atom

(a) P - (ii), Q - (i), R - (iii)

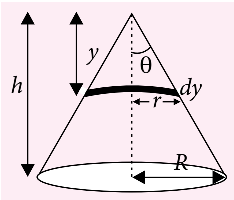
(b) P - (iv), Q - (iii), R - (ii)

(c) P - (i), Q - (iv), R - (iii)

(d) P - (ii), Q - (iv), R - (iii)

SOLUTIONS

1. **(d) :** Let ρ be the density of solid cone.
Consider a disc of radius r, thickness dy at a distance of y from its vertex. Then mass of this disc is
$dm = \rho\pi r^2\, dy$



$$\therefore \quad y_{cm} = \frac{\int_0^h y\, dm}{\int_0^h dm}$$

$$= \frac{\int_0^h \rho\,\pi r^2\, y\, dy}{\int_0^h \rho\,\pi r^2\, dy} \quad \text{...(i)}$$

From figure, $\tan\theta = \frac{r}{y} = \frac{R}{h}$ or $r = \frac{Ry}{h}$ ...(ii)

Putting eqn. (ii) in (i), we get

$$y_{cm} = \frac{\int_0^h \frac{\rho\pi R^2}{h^2} y^3\, dy}{\int_0^h \frac{\rho\pi R^2}{h^2} y^2\, dy}$$

$$= \frac{\int_0^h y^3\, dy}{\int_0^h y^2\, dy} = \frac{\left[\frac{y^4}{4}\right]_0^h}{\left[\frac{y^3}{3}\right]_0^h} = \frac{h^4}{4} \times \frac{3}{h^3} = \frac{3h}{4}$$

$\therefore$ Distance of the centre of mass of a solid uniform cone from its vertex, $z_0 = y_{cm} = \frac{3h}{4}$.

2. **(d) :** Intensity of light, $I = u_{av}c$

Also, $I = \frac{P}{4\pi r^2}$ and $u_{av} = \frac{1}{2}\varepsilon_0 E_0^2$

$$\therefore \quad \frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$\text{or} \quad E_0 = \sqrt{\frac{2P}{4\pi\varepsilon_0 r^2 c}}$$

Here, $P = 0.1$ W, $r = 1$ m, $c = 3 \times 10^8$ m s^{-1}

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N C^{-2} m^2

$$\therefore \quad E_0 = \sqrt{\frac{2 \times 0.1 \times 9 \times 10^9}{1^2 \times 3 \times 10^8}} = \sqrt{6} = 2.45 \text{ V m}^{-1}$$

3. **(c) :** Let length of the pendulum wire be l.

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{l}{g}}$ ...(i)

When an additional mass M is added to bob, let Δl be the extension produced in wire.

Then $T_M = 2\pi\sqrt{\frac{l + \Delta l}{g}}$ ...(ii)

Now, $Y = \frac{\text{stress}}{\text{strain}} = \frac{Mg/A}{\Delta l / l} \Rightarrow \frac{\Delta l}{l} = \frac{Mg}{AY}$ ...(iii)

From eqns. (i) and (ii), we get

$$\frac{T_M}{T} = \sqrt{\frac{l + \Delta l}{l}}$$

$$\text{or} \quad \left(\frac{T_M}{T}\right)^2 = \frac{l + \Delta l}{l} = 1 + \frac{\Delta l}{l} = 1 + \frac{Mg}{AY}$$

(Using (iii))

$$\text{or} \quad \frac{Mg}{AY} = \left(\frac{T_M}{T}\right)^2 - 1 \quad \text{or} \quad \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

4. **(d) :** For a simple pendulum, variation of kinetic energy and potential energy with displacement d is

$$\text{KE} = \frac{1}{2}m\omega^2(A^2 - d^2) \text{ and } \text{PE} = \frac{1}{2}m\omega^2 d^2$$

where A is amplitude of oscillation.

When $d = 0$, $\text{KE} = \frac{1}{2}m\omega^2 A^2$, $\text{PE} = 0$

When $d = \pm A$, KE = 0, $\text{PE} = \frac{1}{2}m\omega^2 A^2$

Therefore, graph (d) represents the variation correctly.

5. **(d) :** Frequency of sound emitted by train,
$\upsilon = 1000$ Hz
Speed of train (source), $v_s = 20$ m s^{-1}
Speed of sound, $v = 320$ m s^{-1}
Observer is stationary.
Frequency heard by person as train approaches him

$$\upsilon_1 = \left(\frac{v}{v - v_s}\right)\upsilon$$

$$= \left(\frac{320}{320 - 20}\right) \times 1000 = \frac{3200}{3} \text{ Hz}$$

Frequency heard by person as train moves away from him

$$\upsilon_2 = \left(\frac{v}{v + v_s}\right)\upsilon = \left(\frac{320}{320 + 20}\right) \times 1000 = \frac{32000}{34} \text{ Hz}$$

∴ Percentage change in frequency

$$= \left(\frac{\upsilon_2 - \upsilon_1}{\upsilon_1}\right) \times 100$$

$$= \left(\frac{\frac{32000}{34} - \frac{3200}{3}}{\frac{3200}{3}}\right) \times 100 \approx -12\%$$

Negative sign implies that the frequency heard by person decreases as the train passes him.

6. (b) : $V = IR$

As $I = neAv_d$ and $R = \frac{\rho l}{A}$

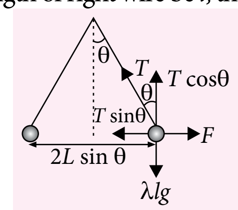
$\therefore \quad V = neAv_d \times \frac{\rho l}{A} \quad$ or $\quad \rho = \frac{V}{nev_d l}$

Here, $V = 5$ V, $n = 8 \times 10^{28}$ m^{-3}, $v_d = 2.5 \times 10^{-4}$ m s^{-1}, $l = 0.1$ m, $e = 1.6 \times 10^{-19}$ C

$$\therefore \quad \rho = \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1}$$

$= 0.156 \times 10^{-4}\ \Omega$ m $\simeq 1.6 \times 10^{-5}\ \Omega$ m

7. (d) : Let the length of right wire be l, then its mass is λl.



Force acting on this wire are tension (T), weight (λlg) and force of repulsion due to other wire (F).

From figure, $T \cos\theta = \lambda lg$ ...(i)

$T \sin\theta = F$ ...(ii)

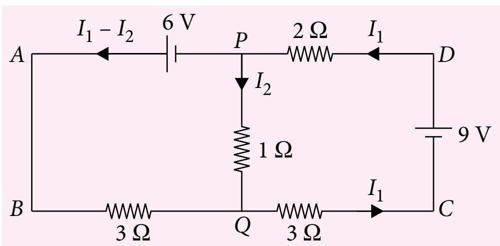
Here, $F = \frac{\mu_0}{2\pi} \frac{I^2 l}{(2L \sin\theta)}$

or $\quad T \sin\theta = \frac{\mu_0}{2\pi} \frac{I^2 l}{(2L \sin\theta)}$ (Using (ii))

or $\quad \frac{\lambda lg}{\cos\theta} \sin\theta = \frac{\mu_0}{2\pi} \frac{I^2 l}{(2L \sin\theta)}$ (Using (i))

$$\Rightarrow \quad I = \sqrt{\frac{4\pi L\lambda g \sin^2\theta}{\mu_0 \cos\theta}} = 2\sin\theta \sqrt{\frac{\pi \lambda g L}{\mu_0 \cos\theta}}$$

8. (a) :



Applying KVL in loop $PQCDP$

$-1I_2 - 3I_1 + 9 - 2I_1 = 0 \Rightarrow 5I_1 + I_2 = 9$ ...(i)

Applying KVL in loop $PQBAP$

$-1I_2 + 3(I_1 - I_2) - 6 = 0 \Rightarrow 3I_1 - 4I_2 = 6$ ...(ii)

Solving eqns. (i) and (ii), we get

$I_1 = 1.83$ A, $I_2 = -0.13$ A

∴ The current in the 1 Ω resistor is 0.13 A, from Q to P.

9. (d) : Given, wavelength of light, $\lambda = 500$ nm $= 500 \times 10^{-9}$ m

Least distance of distinct vision, $D = 25$ cm $= 25 \times 10^{-2}$ m

Radius of pupil, $r = 0.25$ cm

∴ Diameter of pupil, $d = 2r = 0.50$ cm $= 0.50 \times 10^{-2}$ m

Resolving power of eye, $\Delta\theta = \frac{1.22\lambda}{d}$

$$= \frac{1.22 \times 500 \times 10^{-9}}{0.50 \times 10^{-2}} = 1.22 \times 10^{-4} \text{ rad}$$

∴ Minimum separation that eye can resolve,

$x = \Delta\theta D$

$= 1.22 \times 10^{-4} \times 25 \times 10^{-2}$

$= 30.5 \times 10^{-6}$ m $\simeq 30$ μm

10. (b) : When key K_1 is kept closed, a steady current $I_0\left(= \frac{\varepsilon}{R}\right)$ flows through the circuit.

When K_1 is opened and K_2 is closed, current at any time t in the circuit is

$$I = I_0 e^{-t/\tau} = \frac{\varepsilon}{R} e^{-\frac{tR}{L}} \qquad \left(\because \tau = \frac{L}{R}\right)$$

Here, $\varepsilon = 15$ V, $R = 0.15$ kΩ $= 150$ Ω

$L = 0.03$ H, $t = 1$ ms $= 10^{-3}$ s

$$\therefore \quad I = \frac{15}{150} e^{-\left(\frac{10^{-3} \times 150}{0.03}\right)} = \frac{e^{-5}}{10} = \frac{1}{10e^5} = \frac{1}{10 \times 150}$$

$= 6.67 \times 10^{-4}$ A

$= 0.67$ mA

11. (c) : At any time t, the equation of the given circuit is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \quad \text{...(i)}$$

which is equivalent to that of a damped pendulum. The solution to eqn. (i) is

$$q = Q_0 e^{-Rt/2L} \cos(\omega' t + \phi)$$

where $\quad \omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$.

The square of maximum charge on capacitor at any time t is

$$Q_{max}^2 = Q_0^2 \, e^{-Rt/L} \cos^2(\omega' t + \phi)$$

$\therefore$ It decays exponentially with time.
For L_2 $(L_2 < L_1)$, the curve is more steep.

12. (d) : Equivalent capacitance of the circuit

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{(1+2)} = \frac{1}{C} + \frac{1}{3}$$

or $\quad C_{eq} = \dfrac{3C}{C+3}$

Total charge in the circuit, $Q = C_{eq}E = \dfrac{3CE}{C+3}$

Charge on the 2 μF capacitor,

$$Q_2 = \frac{2}{3}Q = \frac{2}{3} \times \frac{3CE}{(C+3)} = \frac{2CE}{C+3}$$

or $\quad Q_2 = \dfrac{2E}{1+\dfrac{3}{C}}$

and $\quad \dfrac{dQ_2}{dC} = \dfrac{6E}{(C+3)^2}$

As C increases, Q_2 increases and slope of $Q_2 - C$ curve decreases. Hence, graph (d) represents the correct variation.

13. (a) : A cube of maximum possible volume is cut from a solid sphere of radius R, it implies that the diagonal of the cube is equal to the diameter of sphere, *i.e.*, $\sqrt{3}\,a = 2R$

or $\quad a = \dfrac{2R}{\sqrt{3}}$

Density of solid sphere, $\rho' = \dfrac{M}{V} = \dfrac{M}{\dfrac{4\pi}{3}R^3}$

$\therefore$ Mass of cube, $M' = \rho V' = \rho a^3$ $(\because \rho = \rho')$

$$= \frac{M}{\frac{4\pi}{3}R^3}\left(\frac{2R}{\sqrt{3}}\right)^3 = \frac{2M}{\sqrt{3}\,\pi}$$

Moment of inertia of the cube about an axis passing through its center and perpendicular to one of its faces is

$$I = \frac{M'a^2}{6} = \frac{1}{6}\frac{2M}{\sqrt{3}\,\pi}\left(\frac{2R}{\sqrt{3}}\right)^2 = \frac{4MR^2}{9\sqrt{3}\,\pi}$$

14. (d) : $T = 2\pi\sqrt{\dfrac{L}{g}} \Rightarrow g = \dfrac{4\pi^2 L}{T^2} = \dfrac{4\pi^2 Ln^2}{t^2}$ $\left(\because T = \dfrac{t}{n}\right)$

Maximum percentage error in g

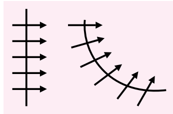
$$\frac{\Delta g}{g}\times 100 = \frac{\Delta L}{L}\times 100 + 2\frac{\Delta t}{t}\times 100$$

$$= \frac{0.1}{20.0}\times 100 + 2\times\frac{1}{90}\times 100$$

$= 2.72\% \simeq 3\%$

$\therefore$ Accuracy in the determination of g is approximately 3%

15. (b): Consider a plane wavefront travelling horizontally.



As refractive index of air increases with height, so speed of wavefront decreases with height. Hence, the light beam bends upwards.

16. (a): Given, $\upsilon_m = 5$ kHz, $\upsilon_c = 2$ MHz $= 2000$ kHz

The frequencies of the resultant signal are

$\upsilon_c + \upsilon_m = (2000 + 5)$ kHz $= 2005$ kHz

$\upsilon_c = 2000$ kHz and

$\upsilon_c - \upsilon_m = (2000 - 5)$ kHz $= 1995$ kHz

17. (*): Since entropy is a state function and the entropy change is independent of the path followed, therefore for both cases

$$\Delta S = \int \frac{dQ}{T} = C\int_{T_1}^{T_2}\frac{dT}{T} = C\ln\left(\frac{T_2}{T_1}\right)$$

Here, $T_1 = 100°C = 373$ K

$T_2 = 200°C = 473$ K

$\therefore \quad \Delta S = C\ln\left(\dfrac{473}{373}\right)$

*None of the given options is correct. If unit of temperatures in question paper were Kelvin, then $\Delta S = C\ln\left(\dfrac{200}{100}\right) = C\ln 2 = \ln 2$ *i.e.*, option (d) would have been correct.

18. (a): According to first law of thermodynamics,

$dQ = dU + dW$

Since the shell undergoes an adiabatic expansion

$\therefore \quad dQ = 0$, *i.e.*, $dU = -dW = -pdV$

or $\quad \dfrac{dU}{dV} = -p = -\dfrac{1}{3}\dfrac{U}{V}$ $\quad \left(\text{Given} : p = \dfrac{1}{3}\dfrac{U}{V}\right)$

$\Rightarrow \quad \dfrac{dU}{U} = -\dfrac{1}{3}\dfrac{dV}{V}$

Integrating both sides

$$\ln U = -\frac{1}{3}\ln V + \ln C$$

or $\quad UV^{1/3} = C \quad$ ...(i)

Given, $u = \dfrac{U}{V} \propto T^4$ or $U = KVT^4$

Putting this in eqn. (i)

$$KVT^4V^{1/3} = C$$
$$T^4V^{4/3} = C/K$$

or $T^4\left(\frac{4\pi}{3}R^3\right)^{4/3} = C/K$ $\quad (\because \ V = \frac{4\pi}{3}R^3)$

$\Rightarrow \ T^4R^4 = C' \Rightarrow T \propto \frac{1}{R}$

19. (a) : Using $h = ut + \frac{1}{2}at^2$

For stone 1, $y_1 = 10t - \frac{1}{2}gt^2$

For stone 2, $y_2 = 40t - \frac{1}{2}gt^2$

Relative position of the second stone with respect to the first, $\Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 - 10t + \frac{1}{2}gt^2$
$= 30\,t$

After 8 seconds, stone 1 reaches ground, *i.e.*, $y_1 = -240$ m

$\therefore \ \Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 + 240$

Therefore, it will be a parabolic curve till other stone reaches ground.

20. (a, b) : Potential on the surface of charged solid sphere

$$V_0 = \frac{Kq}{R}$$

Spherical surface of radius r inside this sphere will be equipotential surface with potential $V(> V_0)$

$$V = \frac{Kq}{2R^3}(3R^2 - r^2) = \frac{V_0}{2R^2}(3R^2 - r^2)$$

$\therefore \ $ For $V = \frac{3V_0}{2}, \ \frac{3V_0}{2} = \frac{V_0}{2R^2}(3R^2 - R_1^2) \Rightarrow R_1 = 0$

For $V = \frac{5V_0}{4}, \ \frac{5V_0}{4} = \frac{V_0}{2R^2}(3R^2 - R_2^2) \Rightarrow R_2 = \frac{R}{\sqrt{2}}$

Spherical surface of radius r' outside this sphere will be equipotential surface with potential $V'(< V_0)$

$$V' = \frac{Kq}{r'} = \frac{V_0R}{r'}$$

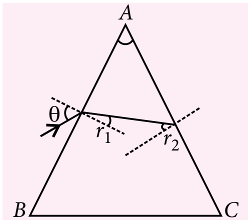
$\therefore \ $ For $V' = \frac{3V_0}{4}; \ \frac{3V_0}{4} = \frac{V_0R}{R_3} \Rightarrow R_3 = \frac{4R}{3}$

For $V' = \frac{V_0}{4}; \ \frac{V_0}{4} = \frac{V_0R}{R_4} \Rightarrow R_4 = 4R$

Here $R_1 = 0$, $R_2 < (R_4 - R_3)$, $2R < R_4$ and
$(R_2 - R_1) < (R_4 - R_3)$

So, options (a) and (b) are correct.

21. (c) : According to Snell's law



$\sin\theta = \mu\sin r_1$

$\Rightarrow \ \sin r_1 = \frac{\sin\theta}{\mu}$

or $\ r_1 = \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$

Now, $A = r_1 + r_2$

$\therefore \ r_2 = A - r_1 = A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$ $\quad$ ...(i)

For the ray to get transmitted through the face AC, r_2 must be less than critical angle,

i.e., $r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$

or $\ A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) < \sin^{-1}\left(\frac{1}{\mu}\right)$ $\quad$ (using (i))

$\Rightarrow \ \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) > A - \sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow \ \frac{\sin\theta}{\mu} > \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)$

$\Rightarrow \ \theta > \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

22. (a) : $I = 12$ A, $\vec{B} = 0.3\,\hat{k}$ T,

$A = 10 \times 5$ cm^2 $= 50 \times 10^{-4}$ m^2

$\vec{M} = IA\hat{n} = 12 \times 50 \times 10^{-4}\,\hat{n}$ A m^2

$= 6 \times 10^{-2}\,\hat{n}$ A m^2

Here, $\vec{M}_1 = 6\times10^{-2}\,\hat{i}$ A m^2, $\vec{M}_2 = 6\times10^{-2}\,\hat{k}$ A m^2

$\vec{M}_3 = -\,6\times10^{-2}\,\hat{j}$ A m^2, $\vec{M}_4 = -\,6\times10^{-2}\,\hat{k}$ A m^2

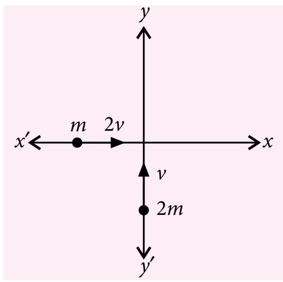
$\vec{M}_2$ is parallel to $\vec{B}$, it means potential energy is minimum, therefore in orientation (2) the loop is in stable equilibrium.

$\vec{M}_4$ is antiparallel to $\vec{B}$, it means potential energy is maximum, therefore in orientation (4) the loop is in unstable equilibrium.

23. (c)

24. (a) : Applying the principle of momentum conservation

$m(2v\hat{i}) + 2m(v\hat{j}) = (m + 2m)\vec{v}'$



$2mv\hat{i} + 2mv\hat{j} = 3m\vec{v'}$

$\Rightarrow \quad \vec{v'} = \frac{2v}{3}(\hat{i} + \hat{j})$

or $\quad v' = |\vec{v'}| = \frac{2\sqrt{2}}{3}v$

Initial energy of the system,

$$E_i = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2$$
$$= 2mv^2 + mv^2 = 3mv^2$$

Final energy of the system, $E_f = \frac{1}{2}(3m)v'^2$

$$= \frac{3m}{2}\left(\frac{2\sqrt{2}}{3}v\right)^2$$
$$= \frac{4}{3}mv^2$$

$\therefore$ Percentage loss in the energy $= \frac{E_i - E_f}{E_i} \times 100$

$$= \frac{3mv^2 - \frac{4}{3}mv^2}{3mv^2} \times 100$$
$$= \frac{5}{9} \times 100 \approx 56\%$$

25. (a) : Average time of collision between molecules,

$$\tau = \frac{\text{Mean free path}(\lambda)}{\text{Mean speed }(\bar{v})} = \frac{1}{\left(\sqrt{2}\pi d^2 \frac{N}{V}\right)\left(\sqrt{\frac{8k_BT}{m\pi}}\right)}$$

$\Rightarrow \quad \tau \propto \frac{V}{\sqrt{T}}$ or $T \propto \frac{V^2}{\tau^2}$ ...(i)

For adiabatic expansion, $TV^{\gamma-1}$ = constant

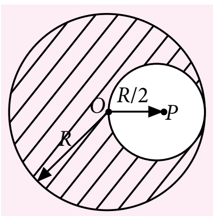
or $\quad \frac{V^2}{\tau^2}V^{\gamma-1}$ = constant

$\Rightarrow \quad \tau \propto V^{\frac{(\gamma+1)}{2}}$

Comparing it with $\tau \propto V^q$, we get $q = \frac{\gamma+1}{2}$

26. (d): Potential at point P (centre of cavity) before removing the spherical portion,

$$V_1 = \frac{-GM}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right)$$
$$= \frac{-GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right)$$
$$= \frac{-11GM}{8R}$$



Mass of spherical portion to be removed, $M' = \frac{MV'}{V}$

$$= \frac{M\frac{4\pi}{3}\left(\frac{R}{2}\right)^3}{\frac{4\pi}{3}R^3} = \frac{M}{8}$$

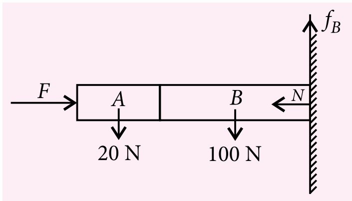
Potential at point P due to spherical portion to be removed

$$V_2 = \frac{-3GM'}{2R'} = \frac{-3G(M/8)}{2(R/2)} = \frac{-3GM}{8R}$$

$\therefore$ Potential at the centre of cavity formed

$$V_P = V_1 - V_2$$
$$= \frac{-11GM}{8R} - \left(\frac{-3GM}{8R}\right) = \frac{-GM}{R}$$

27. (a) : Various forces acting on the system are shown in the figure.



For vertical equilibrium of the system,

f_B = 100 N + 20 N = 120 N

28. (c) : The electric field lines around the cylinder must resemble that due to a dipole.

29. (c) : For an electron in n^{th} excited state of hydrogen atom,

kinetic energy $= \frac{e^2}{8\pi\epsilon_0 n^2 a_0}$

potential energy $= \frac{-e^2}{4\pi\epsilon_0 n^2 a_0}$

and total energy $= \frac{-e^2}{8\pi\epsilon_0 n^2 a_0}$

where a_0 is Bohr radius.

As electron makes a transition from an excited state to the ground state, n decreases. Therefore kinetic energy increases but potential energy and total energy decrease.

30. (a) : Franck-Hertz Experiment - Discrete energy levels of atom.

Photo-electric experiment - Particle nature of light.

Davison-Germer Experiment - Wave nature of electron.

CROSS WORD

Readers can send their answer with complete address before 15th of every month to win exciting prizes. Winners' name with their valuable feedback will be published in next issue.

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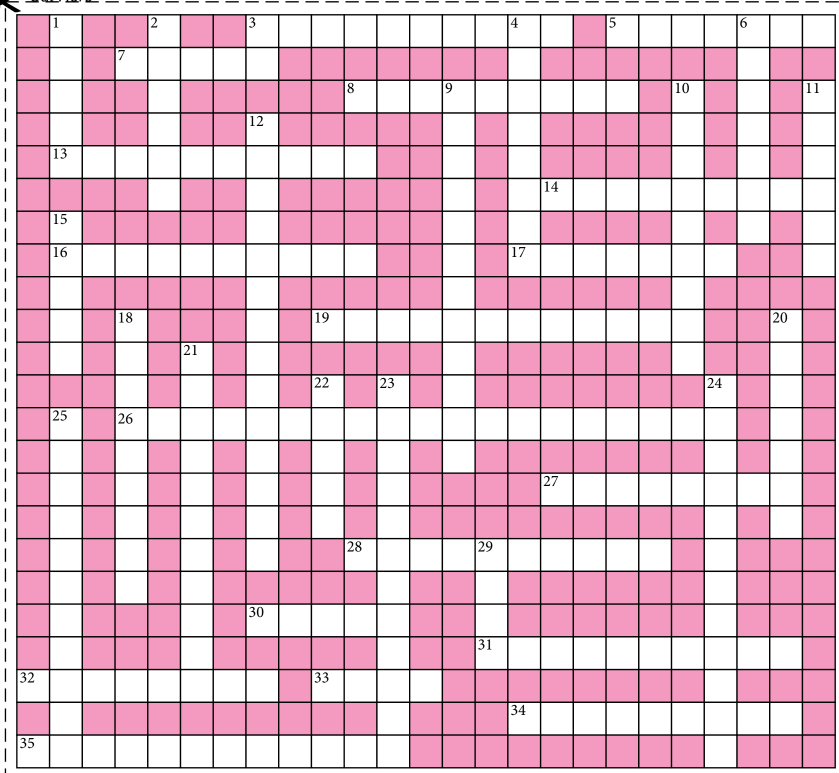

ACROSS

3. A substance that can sustain an electric field and act as an insulator. (10)
5. Atomic unit of energy. (7)
7. A collective name for both hadrons and bosons. (5)
8. A star that explodes and leaves a neutron star remnant. (9)
13. Substance which does not allow the passage of X-rays or other radiations. (10)
14. The study of universe and its contents. (9)
16. An instrument used for measuring the moisture content in the atmosphere. (10)
17. Scale which measures power of an earthquake. (7)
19. The emission of light as a result of the excitation of atoms by a source of energy other than heat. (12)
26. A cataract operation in which the diseased lens is reduced to a liquid by ultrasonic vibrations and drained out of the eye. (19)
27. Process of bonding one metal over another to protect the inner metal from corrosion. (8)
28. Academic study of musical instrument. (10)
30. The hottest planet. (5)
31. A hypothetical form of matter invisible to electromagnetic radiation. (4, 6)
32. A device used to maintain constant pressure in a closed chamber. (8)
33. Smallest unit of matter. (4)
34. An optical instrument for viewing objects that are in an obstructed field of vision. (9)
35. A wave in which each point on the axis of the wave has an associated constant amplitude. (8, 4)

DOWN

1. A device used to detect and find range of an obstacle by using the echo method. (5)
2. SI unit of thermodynamic temperature. (6)
4. A combination of electric or magnetic fields used to capture charged particles in vacuum. (3, 4)
6. An optical phenomena that is caused by reflection, refraction and dispersion of light in water droplets. (7)
9. Two electrons spinning in opposite directions. (8, 4)
10. An instrument for measuring high temperature. (9)
11. Sound producing organ in humans. (6)
12. A region in a magnetic material in which magnetization is in a uniform direction. (8, 6)
15. A class of electron orbits in an atom in which the electrons have the same principal quantum number. (5)
18. Discoverer of the moons of mars. (5, 4)
20. The flow of current in a path other than that intended, due to imperfect insulation. (7)
21. Category under which pluto comes. (5, 6)
22. The shadow cast by the moon onto the earth during a solar eclipse. (5)
23. An instrument for the detection of charge. (12)
24. A reaction in which a particle and its antiparticle collide and disappear, releasing energy. (12)
25. A fluid used as the working fluid of a refrigerator. (11)
29. One of the universal logic gates. (4)